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Full Length Research PapeR

Supply chain planning with scenario based probabilistic demand

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Raw materials ordering policy and the manufacturing lot size for fixed interval deliveries of manufactured products to multiple customers, considered to as a key factor in managing the supply chain logistic more economically whenever probabilistic demand exists. In this paper we developed a weighted probabilistic total cost function in a manufacturing center which supplies manufactured products to multiple customers, with a fixed-quantity at a fixed time-interval to each of the customers while total demand of the customers have a probabilistic behavior and maybe changes during planning horizon. An optimal multi-ordering policy for procurement process of raw materials in a single manufacturing system used to minimize the total acquired cost due to raw materials and manufactured products inventories regarding to different predicted demand alternatives in the production time horizon. The carried over inventory of manufactured products that remained from the previous cycle has been used as initial manufactured products inventory and as a result production schedule shifted ahead for the next cycle. A closed-form solution for the minimal total cost for the entire inventory-production system formulated. The algorithm considered as the solution finding procedure for multiple customer systems with probabilistic future demand rate.

Key words: Raw materials, policy, multiple consumers, demand.

INTRODUCTION

Inventory plays a significant role in many of manufacturing systems. A large number of manufacturing facilities uses for carry large inventories of manufactured products at the supply docks. Newman [Newman, 1988] illustrated that for eliminating any delays in the delivery process when the buyers receive manufactured products based on JIT system. Yilmaz (1992); Parlar and Rempala (1992); Pan and Liao (1989) and Ramasesh (1990) developed optimal ordering policy and quantitative production models in the single-stage production system. Lu (1995) formulated a one-vendor multi-buyer integrated inventory model. Goyal (1995); Goyal and Gupa (1989) and Aderhunmu et al. (1995) have developed some quantitative models for integrated vendor-buyer policy in a just-in-time manufacturing process. Golhar and Sarker (1992); Jamal and Sarker (1993) and Sarker and Parija (1994, 1996) documented some single-product models in a just-in-time production-delivery system. Banerjee (1992) illustrated a lot sizing model with the concentration to the work-in-process in response to periodic and not probabilistic demands. Park and Yun (1984) suggested a stepwise partial enumeration algorithm for solving fluctuating demand problems. Sarker and Parija (1994, 1996) developed the model of Golhar and Sarker, 1992.

To take into account cyclic scheduling for a multi-product manufacturing system, Nori and Sarker, 1996 revised and improved the model of Sarker and Parija, 1996. Robert et al. (2007) considered a two-echelon supply chain. Liang-Yuh et al. (2004) presented a single vendor-single buyer integrated system in which lead time for demand is deterministic and stochastic with permitted shortage. ManMohan et al. (2007) analyzed supply chain system under demand uncertainty with using stochastic programming. Seliaman et al. (2008) considered the case of a three-stage non-serial supply chain system. With the review of the literature this is obvious that limited researches related to optimal ordering and production policies for the manufacturing

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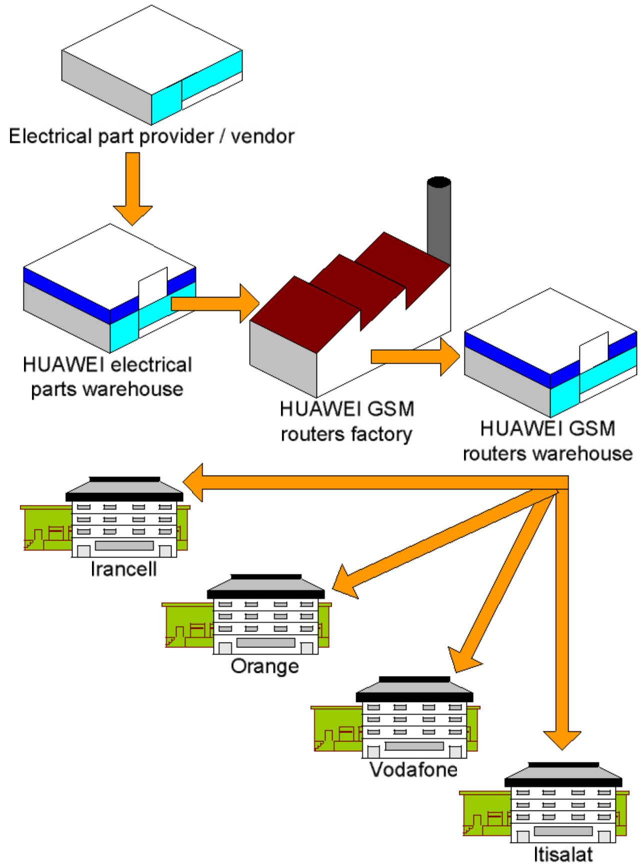

Figure 1. Typical supply chain logistics in a telecommunication company (HUAWEI).

systems with multiple customers, fixed time-intervals and probabilistic customer's demands, has been taken.

Logistic system

Deliveries of the manufactured products with the fixes intervals to multiple customers require a well defined, coordinated and reliable manufacturing system in which supply of raw materials for production and consequently delivery of manufactured products to all of the customers precisely takes over. In this paper we focused to a scenario based production-inventory system such as a supply chain logistics system in telecommunication industries that has been shown in Figure 1. An electrical parts vendor supplies electrical parts to for example HUAWEI company that manufactures GSM network core routers and instruments and also plays a significant role in the telecom industry in the world, which, in turn, are delivered to several outside customers (like GSM operators) such as for example Irancell, Itisalat, Vodafone and Orange. While the total demand of the mentioned customers maybe changes according to their network instruments planning updates. In another word it will have some alternatives or maybe a probabilistic function that marketing will estimate it according to its market analyses tools and techniques. To satisfy buyers demand in the different time-intervals, the manufacturing company (HUAWEI) has to regularly maintain its production rate for procuring electronic parts at regular time intervals. Therefore, both of the manufacturing company and customers need to operate base on a balanced and coordinated logistics, and in order to minimize production-inventory system costs, the supply chain logistics of raw materials (electronic parts) and manufactured products (GSM network routers) should operate as an efficiently integrated system. Also because each of the customer's demand is not fix during production time horizon, and also these changes has a predictable rule or probabilistic function according to marketing and demand observatory studies, therefore we should consider all of the alternatives and possibilities according to its probabilistic existence weights.

Problem definition

A manufacturing company acquire its raw materials via outsourcing process, takes them under process to produce a manufactured product, stores them in a manufactured products inventory, and at last delivers manufactured products to several customers with a fixed quantities and intervals. The annual demands of these customers are not known precisely and have a scenario based behavior but for each of the customers at the beginning of the planning process for each time horizon we can assume that it is a constant parameter. The raw material is non-perishable, and therefore it should be supplied instantaneously to the manufacturing facility. Shortage of manufactured products due to insufficient manufactured products production is not allowed. Demand is base on probabilistic function and according to company policy, instead of demand, average demand for planning time horizon takes into calculations.

In order to minimize the total inventory cost to deliver the products to multiple customers, an optimal ordering policy for raw materials, an economic production run length and an optimal manufacturing batch size as what that developed by Parija and Sarker [17], used in this model. In a single-order policy, raw material required for an inventory cycle are acquired once at the beginning point of an uptime period and the required materials at the later part of a production period brings unnecessary inventory holding cost by being procured in the early part of the cycle. Also a multi-order policy for procuring raw materials could decrease inventory holding cost because this policy encourages the timely use of raw materials. Therefore, in this research, the feasibility and other relevant appropriate aspects of such a logistic system has been considered.

In this paper, Total cost function to determine both optimal-order policy and optimal batch size for a production run and to minimize the total inventory costs for both raw materials and manufactured products has been used (Sarker and Parija, 1999). Also we consider opportunity cost for situations that we will interface to more demands in front of what that we estimated before and extra sales costs whenever that we have less demand rather than what that we predicted before. These two added costs to the model considered due to determining more demand alternatives and to raise the model compatibility with the real business dynamic environments. The supply chain system is described in section 2. A total cost model for the manufacturing system is developed in section 3. A solution algorithm for this model explained in section 4. Finally, conclusions stated in the last section.

THE SUPPLY CHAIN SYSTEM

To find an Economic Order Quantity (EOQ) for the raw materials and an Economic Manufacturing Quantity (EMQ) for the production run, two types of inventory holding costs are considered (Sarker and Parija, 1999): raw materials holding cost, H_R and manufactured products holding cost, H_F. Order costs includes the ordering cost of the raw materials, K_R and the manufacturing setup cost for each batch, K_S. Opportunity cost, OC_F and extra sales cost, EC_F considered for illustrating upper and lower demand situations costs. α, β, γ considered as demand probabilities for three different states of the customer's demand. Following notation is used to model the system:

Definitions and notation

To model the relationship of the entire system of raw materials and manufactured products demands, the following notation is used:

Raw material related: D_R, H_0, K_0, n, Q_0 and $\bar{Q}_R$.

Manufactured products related: D_F, $D_{i,F}$, f, H_F, K_S, m_i, P, $\bar{Q}_{avg}$, Q_F, $\bar{Q}_M$, $\bar{Q}_S$, x_i, Y, OC_F, EC_F, $E(D_F)$, α, β, γ, $F(D_F)$.

Cycle time related: L_i, T, T_0, T_1 and T_2

Definitions:

$\bar{D}_F$ = total demand for manufactured products by all customers, units/year;

$D_{i,F}$ = total demand for manufactured products by customer i, units/year;

$\bar{D}_R$ = total demand for raw materials by the production facility, units/year;

$\bar{f}$ = conversion factor of the raw materials to manufactured products, $f = D_R/D_F$;

H_F = holding cost of manufactured products, \$/unit/year;

H_R = holding cost of raw materials, \$/unit/year;

K_R = ordering cost of raw materials, \$/order;

K_S = manufacturing setup cost per batch, \$/batch;

L_i = given time between successive shipments of manufactured products to customer i $(i=1,\ldots,N)$;

m_i = number of full shipments of manufactured products to customer i per cycle time T;

n = number of orders of raw materials during the uptime T_1;

P = production rate, units/year;

Q_{avg} = average inventory of manufactured products per cycle, in units;

Q_F = quantity of manufactured products manufactured per setup, units/batch;

$\bar{Q}_F$ = quantity of manufactured products inventory held at the end of uptime T_1 , in units;

$Q_F(t)$ = manufactured products inventory on hand at time t ; $Q_F(t) = Q_M(t) - Q_S(t)$

$Q_M(t)$ = quantity of manufactured products in the inventory at time t , in units;

Q_0 = quantity of raw materials ordered each time, units/order; $Q_0 = Q_R/n$;

Q_R = quantity of raw materials required for each batch; $Q_R = \frac{Q_F}{f} = nQ_0$;

Q_S = total quantity of manufactured products shipped, in units/cycle;

$Q_S(t)$ = total quantity of manufactured products shipped by time t ;

T = cycle time $T = \frac{Q_F}{D_F} = m_i L_i$, While {1,...,N};

T_{0n} = production start time;

T_1 = manufacturing period (uptime); $T_1 = \frac{Q_F}{P}$;

T_{2} = downtime; $T_2 = T - T_1 = Q_F\left(\frac{1}{D_F} - \frac{1}{P}\right)$;

x_i = quantity of manufactured products shipped to customer i at a fixed interval of time L_i, units/shipment;

$x_i = \frac{Q_F}{m_i} = L_i D_F$, While $i \in \{1, ..., N\}$;

Y = quantity produced during L_i period; $Y = L_i P = \frac{x_i P}{D_F}$;

$Y - x_i = (\frac{P}{D_F} - 1)x_i$,

OC_F = opportunity cost for sale manufactured products when demand for manufactured products (D_F) is more than it's calculated average ($E(D_F)$), $/unit;

EC_F = extra sale costs for sale manufactured products when demand for manufactured products (D_F) is less than it's calculated average ($E(D_F)$), $/unit;

$E(D_F)$ = average total demand for manufactured products by all customers, units/year;

$F(D_F)$ = customer's total demand probabilistic function;

α = total customer's demand probability when manufactured products demand (D_F) is equal to average total demand for manufactured products $E(D_F)$, $(D_F = E(D_F))$, 0≤α≤1;

β = total customer's demand probability when manufactured products demand (D_F) is less than average total demand for manufactured products $E(D_F)$, $(D_F \leq E(D_F))$, 0≤β≤1;

γ = total customer's demand probability when manufactured products demand (D_F) is more than average total demand for manufactured products $E(D_F)$, $(D_F \geq E(D_F))$, 0≤γ≤1;

α + β + γ = 1.

The supply chain system

Manufactured products inventory in this model, doesn't have equal behavior comparison with traditional economic batch quantity model with continuous demand (Sarker and Parija, 1996). As depicted in Figure 2, to discourage the undesirable inventory build up, raw materials in this model are ordered n times during the uptime. Because production rate, P, is considered to be higher than the consumption rate, the inventory will keep on building while the production (or uptime) continues. A fluctuating-demand (fixed quantity) of x_i units of manufactured products at the end of every L_i time units to customer i , $(i = 1,2, ..., N)$, is imposed to the manufacturer. This fluctuating demand decreases the manufactured products inventory buildup instantaneously by x_i that makes as a result, an inventory buildup in an increasing triangular fashion during the production period T_1 . x_i units of manufactured products, to satisfy the demand of customer i at an interval of L_i time units, are delivered instantaneously that as a result remains $Y - X_i$ units at hand, where $Y = L_i P$, the quantity that produced during L_i time units at the rate of P units per unit-time. The delivery schedules of manufactured products and quantities shipped to customer i , $(i = 1,2, ..., N)$, are also shown in Figure 2. For N customers $L_1 \leq L_2 \leq \cdots \leq L_{N-1} \leq L_N$ and all x_i, $(i = 1,2, ..., N)$,may notbe equal and m_i, $(i = 1,2, ..., N)$ the number of full shipments to customer i , is a non increasing set of integer numbers $(m_1 \geq m_2 \geq \cdots \geq m_{N-1} \geq m_N)$ such that $m_1 L_1 = m_2 L_2 = \cdots = m_{N-1} L_{N-1} = m_N L_N = T$. The on-hand manufactured products inventory at any time t is the manufactured products produced by that time minus the total inventory delivered to all customers by time t. the on-hand manufactured products at the end of uptime period T_1 , Q_H, decrease instantaneously by x_i units at a regular interval of L_i time units (after the production run) till the end of the last shipment in a cycle are carried over to the next cycle, resulting in a shifted production schedule as reflected in Figure 2 (Sarker and Parija, 1999).

To minimize total cost of inventory, production, loss of

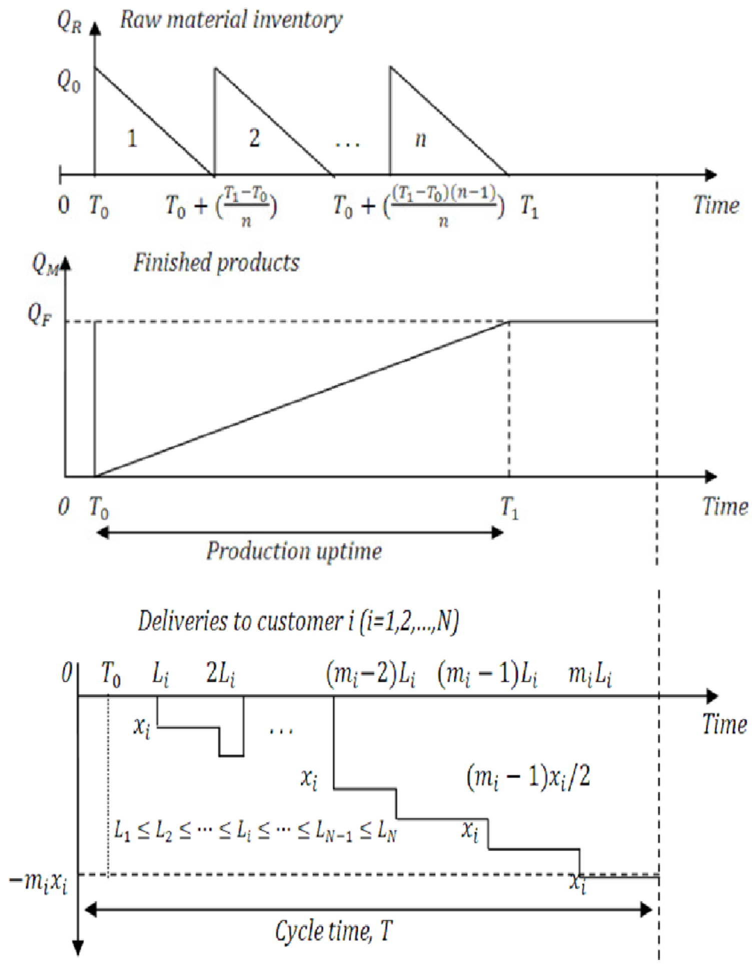

Figure 2. Raw materials and manufactured products inventory.

opportunity and extra sale costs, in this model we have following distinct goals:

(a) Production start time determination
(b) Raw material ordering policy determination
(c) Batch size of manufactured products determination
(d) Opportunity lost costs determination
(e) Extra sales costs determination
(f) Make a tradeoff between different costs of the system to minimize total costs of the entire system.

PRODUCTION COST MODEL

Manufactured products buyers (in this mode GSM operators) usually require these products at specific time intervals and these time intervals are variable for different customers, and also there could be a numerous of like this time intervals in which as a result there will be many deliveries for the manufacturer. However, because of difficult handle of like these customers time intervals, the manufacturing company (in this example HUAWEI) could be not agree with there time intervals; so the manufacturer has an option of giving a fixed set of deliveries which could define according to negotiation process with the customers. Neither of both customer and manufacturer has absolute control on selecting the product delivery frequency. Thus, we assume that $L_1 \le L_2 \le \dots \le L_{N-1} \le L_N$ and to be more confident in case of materials availability and proper delivery, we further assume that $Q_F = P\ (T_1 - T_0) = D_F = \sum_{i=1}^{N} m_i x_i$ and $T = m_i L_i$, While $i \in \{1, 2, \dots, N\}$.

Total cost functions

As depicted in Figure 1, raw material and manufactured products costs are two principal costs of this model, also opportunity and extra sale costs are another types of costs that exist depend on manufactured products demand's states. In the model we assume that total future demand for manufactured products is equal to

demand probability function expected value $E(D_F)$.
System has costs as below:

Raw material costs: In an inventory state where $Q_F = E_i(T_1 - T_0) = D_F = \sum_{i=1}^{N} m_i x_i$ there might be some manufactured products hands over to the next cycle after $\sum_{i=1}^{N} m_i$ shipments to all customers. Therefore according to instantaneous replenishment policy the raw material, any of inventory should not be procured and also the production should not start in the next cycle until the handed-over inventory from the previous cycle, ($Q_F - \sum_{i=1}^{N} m_i x_i$), is used up at the first of the next cycle.

Assume that the procurement of a new inventory of raw materials arises at time T_0 in the next cycle, while T_0 is equal to the required time to use the carried-over inventory from the previous cycle. So the new manufacturing uptime continues for $(T_1 - T_0)$ time units after which the production of manufactured products stops at the peak of the inventory. A series of fluctuating deliveries of manufactured products inventory continues after T_1 until the cycle repeats same as what that done before.

Thus the total cost of raw materials, as stated with Sarker and Parija, 1996 is given by

$$TC_R = \left(\frac{D_R}{Q_0}\right) K_0 + \frac{Q_0 (T_1 - T_0)}{2T} H_0. \quad (1)$$

Now, if there are n replenishments of raw materials during the uptime period $[T_0, T_1]$, and m_i transports of manufactured products of size x_i to customer i $(i = 1,2, \ldots, N)$, during the cycle time T, then, for Q_R, each batch required raw materials, we can write $Q_0 = Q_R/n$ and $Q_F = (T_1 - T_0)P = T\ D_F = T \ldots \sum_{i=1}^{N} m_i x_i$, therefore equation (1) could rewrite as below:

$$TC_R = \left(\frac{nD_R}{Q_R}\right) K_0 + \frac{1}{2}\left(\frac{Q_R}{n}\right)\left(\frac{D_F}{P}\right) H_0. \quad (2)$$

Manufactured products costs: Because of probabilistic changes that exist in the demand of manufactured products, three different demand states maybe arises during the planning time horizon. Total demand may be equal, less or more than what that predicted in the time in which planning done in it. In this model we assumed that total demand may have a probabilistic behavior indeed of each customer's demand, while each customer's demand could have probabilistic behavior as same as what that we considered about total demand in this model. However we assumed that the goal in our model is to minimize total cost of the entire system and there is no demand priority or behavior difference between the customers of the finished goods (for example in our model there is no difference between Irancell, Vodafone and Orange demands behavior and if there is a demand more than what that we predicted before, it's not important which of the customers does it refers to). Three demand states for production system are as belows:

$$D_F = E(D_F)$$

When manufactured products demand (D_F) is equal to expected value for manufactured products demand $(E(D_F))$, we have this state for system costs (when $D_F = E(D_F)$). This state for total manufactured products demand in the model will arise with probability equals to α.

Since $\frac{D_R}{Q_R} = \frac{D_F}{Q_F}$ and $Q_F = f Q_R$ for a conversion factor (or production process efficiency) of raw materials to manufactured products, f, the total cost for manufactured products inventory for average inventory Q_{avg} at a holding cost H_F \$/unit/year can also be written as (Sarker and Parija, 1999)

$$TC_F = \frac{D_F}{Q_F} K_S + Q_{avg} H_F. \quad (3)$$

And the entire cost of the system may be expressed as

$$TC_1(Q_F, n) = \left(\frac{nD_R}{Q_R}\right) K_0 + \frac{D_F}{Q_F} K_S + \frac{1}{2}\left(\frac{Q_R}{n}\right)\left(\frac{D_F}{P}\right) H_0 + Q_{avg} H_F. \quad (4)$$

Therefore, with replacing Q_{avg} in equation (4) with the results from equation (A5) in appendix A, we have (Sarker and Parija, 1999)

$$TC_1(Q_F, n) = \left(\frac{nD_R}{Q_R}\right) K_0 + \frac{D_F}{Q_F} K_S + \frac{1}{2}\left(\frac{Q_R}{n}\right)\left(\frac{D_F}{P}\right) H_0 + \left(\frac{\left(1 - \frac{D_F}{P}\right) Q_F}{2} + \frac{\sum_{i=1}^{N} x_i}{2} + T_0 D_F\right) H_F. \quad (5)$$

$$D_F \leq E(D_F)$$

When manufactured products demand (D_F) is less than expected value for manufactured products demand $(E(D_F))$, we have this state for system costs (when $D_F \leq E(D_F)$). This state for total manufactured products demand in the model will arise with probability equals to β.

In this state manufactured products costs could formulate as below:

$$T_1C_F = \frac{D_F}{Q_F} K_S + (Q_F/2 ...) H_F + EC_F(E(D_F) - D_F) \quad (6)$$

Therefore, replacing $C_{inventory}$ in equation (6) with the results from equation (A5) in appendix A and with regarding raw material costs from equation (2) in section 3.1.1 we have

$$TC_2(Q_F,\pi) = \left(\frac{\pi D_E}{Q_E}\right) K_0 + \frac{D_F}{Q_F} K_S + \frac{1}{2}\left(\frac{Q_E}{\pi}\right)\left(\frac{D_F}{P}\right) H_0 + \left(\frac{\left(1-\frac{D_E}{P}\right)Q_F}{2} + \frac{\sum_{i=1}^{N} x_i}{2} + T_0 D_F\right) H_F + EC_F(E(D_F) - D_F). \quad (7)$$

$$D_F \geq E(D_F)$$

When manufactured products demand (D_F) is more than expected value for manufactured products demand ($E(D_F)$) , we have this state for system costs (when $D_F \geq E(D_F)$). This state for total manufactured products demand in the model will arise with probability equals to γ.

In this state manufactured products costs could formulate as below:

$$T_1C_F = \frac{D_F}{Q_F} K_S + (Q_F/2 ...) H_F + EC_F(E(D_F) - D_F) \quad (8)$$

Therefore, replacing $C_{inventory}$ in equation (8) with the results from equation (A5) in appendix A and with regarding raw material costs from equation (2) in section 3.1.1 we have

$$TC_3(Q_F,\pi) = \left(\frac{\pi D_E}{Q_E}\right) K_0 + \frac{D_F}{Q_F} K_S + \frac{1}{2}\left(\frac{Q_E}{\pi}\right)\left(\frac{D_F}{P}\right) H_0 + \left(\frac{\left(1-\frac{D_E}{P}\right)Q_F}{2} + \frac{\sum_{i=1}^{N} x_i}{2} + T_0 D_F\right) H_F + OC_F(D_F - E(D_F)). \quad (9)$$

As mentioned, three different total cost functions as three different scenarios exists for different states of the entire system base on its total manufactured products demands

Now, if T_0 is a time such that $T_0 \leq T_0'$, then T_0 must satisfy (Sarker and Parija, 1999)

$$(T' - T_0)P \geq \{\sum_{i=1}^{N} [T'/L_i] x_i\} \text{ While } T' \geq T_0 \quad (10)$$

That is, the quantity of manufactured products produced in the time interval $[T_0, T']$ is sufficient to meet the demands (shipments) for each individual customer until the time T', at all times T', after the product begins at T_0.

Also, note that since the first shipment occurs L_i time units after T_0 satisfies $L_i \leq T' - T_0$. Once T_0 has been defined and its role in the model is justified, we next present a result that helps compute T_0:

Theorem 1 (Sarker and Parija, 1999).

If $T_0 = \min \left\{T' - \sum_{i=1}^{N} m_i \left(\frac{x_i}{P}\right)\right\}$

Subject to $T' \geq L_i$, $T'/L_i - m_i \geq T'/L_i - 1$, for $i = 1, 2, \ldots, N$, and T' real, m_i integer, then T_0 satisfies the inequality (10). (11)

Proof. Appendix B

The lower bound on the integer variable m_i will be very useful in actual computations. In theorem 1, illustrated that if T_0 is chosen as stated, then meeting demands for all the products will guarantee.

Start time determination (Sarker and Parija, 1999)

The production start time, T_0, can be determined by solving the problem:

(PST): $Max T_0$,

Subject to

$$(T' - T_0)P \geq \{\sum_{i=1}^{N} [T'/L_i] x_i\}, \text{ While } T' \geq T_0 \geq 0.$$

(12)

This means that we would like to delay starting the procurement and the production until it is absolutely necessary. The idea is to start the production at a time, just in time, to meet the shipment demands so that we do not carry excessive inventory. The constraint in problem (PST) implies that, if the quantity of manufactured products produced in the interval $[T_0, T']$ is $(T' - T_0)P$;

$\left[\frac{T'}{L_i}\right]$ is the number of shipments for customer i until time, T'; and

$\left[\frac{T'}{L_i}\right] x_i$ is the total shipment quantity to customer i until time T',

Then the quantity produced in the interval $[T_0, T']$ is enough to meet the demands for all the customers until time T'. Note that the constraint in (PST) can be written as

$$T_0 \le \min\left\{T' - \left(\frac{\sum_{i=1}^{N} m_i x_i}{P}\right) \mid T' \ge m_i L_i \ge \left[\frac{T'}{L_i}\right] [illegible] \right\} \qquad (13)$$

Which is equivalent to the inequality

$$T_0 \le \min\left\{T' - \left(\frac{\sum_{i=1}^{N} m_i x_i}{P}\right) \mid \frac{T'}{L_i} \ge m_i \ge \frac{T'}{L_i} - 1, T' \ge 0, m_i \ge 0 \text{ and integer}\right\} \qquad (14)$$

The right-hand side of inequality (13) is a Mixed-Integer Programming (MIP) problem. Therefore, the upper bound on T_0 is the solution of the MIP in inequality (13). Hence, the optimal production start time T^*_0 is given clearly by

$$T^*_0 = \min\left\{T' - \left(\frac{\sum_{i=1}^{N} m_i x_i}{P}\right) \mid \frac{T'}{L_i} \ge m_i \ge \frac{T'}{L_i} - 1, T' \ge 0, m_i \ge 0 \text{ and integer}\right\} \qquad (15)$$

Proof for equality 14 given in appendix C.

SOLUTION ALGORITHM

The results illustrated in the previous section used as a platform to solve this production- inventory model for multiple customers with probabilistic demand for manufactured products. Algorithm 1 as a strategy to arrive at a feasible solution for this problem could be use.

Algorithm 1: solution algorithm

Step 1. Compute T_0 by solving the MIP in (15)

Step 2. [illegible] $TC_1(Q_F, \pi)$, $TC_2(Q_F, \pi)$, $TC_3(Q_F, \pi)$.

Step 3. Obtain probabilistic weighted total cost function ($PWTC(Q_F, \pi)$):

$PWTC(Q_F, \pi) = (\alpha TC_1(Q_F, \pi) + \beta TC_2(Q_F, \pi) + \gamma TC_3(Q_F, \pi)) / (TC_1(Q_F, \pi) + TC_2(Q_F, \pi) + TC_3(Q_F, \pi))$ (15)

Step 4. Minimize $PWTC(Q_F, \pi)$ in order to obtain (Q^*_F, π^*)

Step 5. Obtain a pragmatic solution:

Generate two feasible integer solutions in the neighborhood of (Q^*_F, π^*).

$Q^0_F = \max\{\sum_{i=1}^{N} m_i x_i \mid m_i L_i = \text{constant for all } i, \sum_{i=1}^{N} m_i x_i \le [Q^*_F], m_i\text{:integer}\}$

,

$Q^1_F = \max\{\sum_{i=1}^{N} m_i x_i \mid m_i L_i = \text{constant for all } i, \sum_{i=1}^{N} m_i x_i \ge [Q^*_F], m_i\text{:integer}\}$

,

π^0 = Integer n such that n minimizes $|(Q^0_F/\pi) - (Q^*_F/\pi^*)|$, and

π^1 = Integer n such that n minimizes $|(Q^1_F/\pi) - (Q^*_F/\pi^*)|$.

(b) Choose the better of the two candidate solutions:

Optimum (Q_F, π) = (Q^k_F / π^k) where k = arg min $\{PWTC(Q^i_F / \pi^i)$, i = {0,1}}

Conclusions

In the current dynamic industrial environment most of the companies are in the direct interface with probabilistic inventory and production factors. Both the manufacturing company and the customers of the manufactured products need to operate in a harmonic and coordinated logistic, and in order to keep the production-inventory system operative at minimal cost, the supply chain logistics of raw materials and manufactured products should be balanced efficiently. The situation that developed in this paper was about demand behavior. In the previous research demand considered as a fix parameter that we manufacturer efforts to balance its raw materials and manufactured products inventory to achieve minimum total cost for the entire system but in this paper we focused on the behavior of demand and assumed that demand has a probabilistic scenario based behavior and for each alternative we have an specified total cost function that the goal of the model is to minimize the probabilistic weighted total cost function for the entire system regarding to all alternatives for manu-factured products demands.

Future researches can be directed to considering more probabilistic sub-systems and factors in the different combinations of inventory-production-sale or value chain systems.

APPENDIX A (Sarker and Parija, 1999)

Average finished goods inventory

The finished goods inventory level at any time t, as depicted in Figure 2, may be partitioned into the quantity manufactured by time t and the total inventory consumed by the time. At any time t, the level of the finished goods inventory, $\bar{Q}(t)$, is the excess of manufacturing quantity $Q(t)$, over the shipped quantity, $Q(t)$. In order to find the average inventory per cycle, Q_{avg}, the following term needs to be evaluated:

$$Q_{avg} = 1/T \int_0^T Q(t)dt = 1/T \int_0^T Q_M(t)dt - \int_0^T Q_S(t)dt$$

$$= 1/T(\int_0^{T_0} 0dt + \int_{T_0}^{T_1} P(t)dt + \int_{T_1}^{T} P(T_1 - T_0)dt)$$

$$- 1/T \sum_{i=1}^{N} (\int_0^T Q^i_S(t)dt) ,$$

$$= 1/T \{ P(T_1 - T_0)^2/2 + P(T_1 - T_0)(T - T_1)\} - \sum_{i=1}^{N} (1/T \int_0^T Q^i_S(t)dt) ,$$

$$= Q_F(1 - D_F/2P) - T_0 D_F - \sum_{i=1}^{N} (1/T \int_0^{T=m_i L_i} Q^i_S(t)dt). \quad \text{(A2)}$$

From Fig. 2, the total quantity of finished goods shipped to customer i per cycle is given by

$1/T \int_0^{T=m_i L_i} Q^i_S(t)dt = 1/T \; m_i L_i \sum_{k=1}^{m_i} (k-1) x_i L_i = 1/m_i L_i \; m_i (m_i - 1)x_i L_i/2 = (m_i - 1)x_i/2$ (A3)

Hence, considering total shipments to all customers, the average inventory Q_{avg} in Equation (A2) is obtained

$$Q_{avg} = Q_F(1 - D_F/2P) - T_0 D_F - \frac{\sum_{i=1}^{N} (m_i - 1)x_i}{2} \quad \text{(A4)}$$

$$= Q_F(1 - D_F/2P) - ½(\sum_{i=1}^{N} m_i x_i - \sum_{i=1}^{N} x_i) - T_0 D_F,$$

$$= Q_F(1 - D_F/2P) - ½(Q_F - \sum_{i=1}^{N} x_i) - T_0 D_F,$$

$$= Q_F(1 - D_F/P)/2 + \sum_{i=1}^{N} x_i/2 - T_0 D_F, \quad \text{(A5)}$$

Appendix B (Sarker and Parija, 1999)

Theorem 1 proof

$T_0 \leq \{T' - \sum_{i=1}^{N} m_i (\frac{x_i}{P})\}, T' \geq L_1, m_i L_i \leq T', m_i$ integer;

Or, $T_0 \leq \left\{T' - \sum_{i=1}^{N} [T'/L_i] \left(\frac{x_i}{P}\right)\right\}$, While $T' \geq L_1$ $m_i = [T'/L_i] \leq T'/L_i$, is an integer;

Or, $(T' - T_0) \geq \sum_{i=1}^{N} [T'/L_i] \left(\frac{x_i}{P}\right)$, While $T' \geq L_1$;

Or, $(T' - T_0)P \geq \sum_{i=1}^{N} [T'/L_i] x_i$, While $T' \geq L_1$;

Or, $(T' - T_0)P \geq \sum_{i=1}^{N} [T'/L_i] x_i$, While $T' \geq T_0$, because $L_1 \geq T_0$.

Hence, inequality (11) is satisfied.

$$T^*_0 = \text{Min}\left\{T' - \left(\frac{\sum_{i=1}^{N} m_i x_i}{P}\right) \mid \frac{T'}{L_i} \geq m_i \geq \frac{T'}{L_i} - 1, T' \geq 0, m_i \geq 0 \text{ and integer}\right\}$$

APPENDIX C (Sarker and Parija, 1999)

Claim 1. In (MIP), always $m_i^* = [\frac{T'^*}{L_i}]$ for all i .

Proof. Let $(T',m')^*$ be an optimal solution to the MIP problem. Therefore, $T'^* - \sum_{i=1}^{N} m_i (x_i/P) \leq T' - \sum_{i=1}^{N} m_i (x_i/P)$ While $m_i \leq \frac{T'}{L_i}$ and $m_i \geq 0$, integer. By way of contradiction, suppose for some i ∈ *F and F is equal or subset of* {1,…,n}, $m_i^* = [\frac{T'^*}{L_i}] - 1$. Suppose (T'', m'') is a solution to the problem such that $T'' = T'^*$, $m_i'' = [\frac{T'^*}{L_i}]$ for all $i \in F$.

now $m_i'' \geq 0$ and integer; hence, $m_i'' < \frac{T''}{L_i}$ for all i. Clearly, $m_i'' > m_i^*$ for $i \in F$, Hence,

$\sum_{i \in F} m_i' \left(\frac{x_i}{P}\right) > \sum_{i \in F} m_i^* \left(\frac{x_i}{P}\right)$,

Or $\sum_{i=1}^{N} m_i' \left(\frac{x_i}{P}\right) > \sum_{i=1}^{N} m_i^* \left(\frac{x_i}{P}\right)$ since F with $F^c = N$,

$T'' - \sum_{i=1}^{N} m_i'(x_i/P) < T'^* - \sum_{i=1}^{N} m_i^*(x_i/P)$, or

because $T'' = T'^*$.

Therefore (T'', m'') is not optimal. Hence, the proof is immediate.

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