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Full Length Research Paper

Impact of pressure work and heat generation / absorption on free convection flow from a vertical circular cone with variable surface heat flux

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Impact of pressure work and heat generation/absorption on free convection flow from a vertical circular cone with variable surface heat flux is considered. The governing system of partial differential equations is reduced to a system of ordinary differential equations. Mathematica program has been used to solve such system. Comparison of the numerical results is made with previously published results under the special cases, the results are found to be in a good agreement. The solutions are presented in terms of local skin-friction, local Nusselt number, velocity and temperature profiles for values of Prandtl number, pressure work parameter, heat generation/absorption parameter and heat flux gradient parameter.

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Key words: Laminar boundary layer, heat transfer, heat generation / absorption, vertical cone.

INTRODUCTION

Laminar free convection flow and heat transfer over a heated vertical surface is encountered in variety of engineering applications including thermal insulating, cooling of metallic surfaces in a bath and heat dissipation from electronic components and also geophysical fluid dynamics. Numerous authors have investigated laminar free convection flows, especially in the case of non-uniform surface temperature. Mark and Prins (1953, 1954) developed the general relations for similar solutions on isothermal axisymmetric forms and showed that for the flow past a vertical cone has such a solution. The free convection similarity flows about two dimensional axisymmetric bodies with closed lower ends has been studied by Broun et al. (1961). Similarity solutions for free convection from the vertical cone have been exhausted by Hering and Grosh (1962). The study of Hering and Grosh (1962) has been extended by Roy (1974) for the case of high values of the Prandtl number. The laminar free convection flow from a vertical circular cone maintained at non-uniform surface temperature with suction and pressure work has been studied by Alim et al. (2006). Alam et al. (2007) investigated the laminar free convection flow from a vertical permeable circular cone maintained at non-uniform surface temperature with pressure work. The effect of heat generation/absorption on free convective has been studied by scientists and technologists (Chamkha, 2000; Molla et al., 2004; Hady et al., 2006; Molla et al., 2006; Bagai, 2004). In this analysis, we consider (as, Vajravelu and Hadjinolaou, 1993) that the volumetric rate of heat generation should be the volumetric rate of heat generation=

$$\begin{cases} Q_0(T-T_\infty) & for \quad T \ge T_\infty \\ 0 & for \quad T \le T_\infty \end{cases}$$

where Q_0 isthe heat generation or absorption constant.

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In the present study, we have investigated the effect of pressure work and heat generation/absorption on free convection flow from a vertical circular cone with variable surface heat flux.

MATHEMATICAL FORMULATION

A steady two-dimensional laminar free convection flow past a vertical cone with variable surface heat flux is considered. The physical coordinates (x, y) are chosen such that the origin of the coordinates is placed at the vertex of the cone, where the coordinate along the surface of the cone is measured from the origin and y is the coordinate normal to the surface of the cone. The coordinate system and flow configuration are shown in Figure 1. The fluid properties are assumed to be constant.
1-constant. $Q_0(x)$ proportional inversely to x^2, that is, $Q = x^2 Q_0$ and Q is constant).

The boundary layer equations for steady laminar boundary layer along a vertical cone (Alim et al., 2006, 2007) are given below:

$$\frac{\partial(ur)}{\partial x} + \frac{\partial(vr)}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + g\beta\cos\gamma(T - T_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{T\beta u}{\rho C_p}\frac{\partial p}{\partial x} + \frac{\alpha Q}{k}(T - T_\infty) \tag{3}$$

The boundary conditions are as follows:

$$u = 0,\ v = 0,\ q = -k\frac{\partial T}{\partial y} \text{ at } y = 0 \tag{4}$$

$$u = 0,\ T = T_\infty \quad as \quad y \to \infty.$$

The continuity equation can be satisfied by introducing the stream function ψ such that

$$u = \frac{1}{r}\frac{\partial\psi}{\partial y} \text{ and } v = -\frac{1}{r}\frac{\partial\psi}{\partial x} \tag{5}$$

Applying the following transformations:

$$\psi = \upsilon r(Gr_x)^{1/5} f(\eta),\ T - T_\infty = \frac{q_w x}{k}(Gr_x)^{-1/5}\theta(\eta),\ \eta = \frac{y}{x}(Gr_x)^{1/5}$$

$$r = x\sin\gamma,\ Gr_x = \frac{g\beta\cos\gamma q_w x^4}{k\upsilon^2},\ q_w = x^n \tag{6}$$

Substituting the transformations given in (6) into (1)-(4), we obtained the following equations:

$$f''' + \left(\frac{n+9}{5}\right)ff'' - \left(\frac{2n+3}{5}\right)f'^2 + \theta = 0 \tag{7}$$

$$\theta'' + \Pr\left[\left(\frac{n+9}{5}\right)f\theta' - \left(\frac{4n+1}{5} + \in\right)f\theta + \delta\theta\right] = 0 \tag{8}$$

The corresponding boundary conditions to be satisfied are:

$$f = f' = 0,\ \theta = -1 \text{ at } \eta = 0$$
$$f' = 0,\ \theta = 0 \text{ as } \eta \to \infty \tag{9}$$

Here, the primes denotes the differentiation with respect to η. $\Pr = \upsilon/\alpha$ is Prandlt number, $\delta = \alpha Q(Gr_x)^{-\frac{2}{5}}/\upsilon k$ is the heat generation/absorption parameter and $\in = g\beta x / C_p$ is the pressure work parameter which is first used by Gebhart (1962). The equations (7) and (8) subjected to the boundary conditions (9) are solved by using Mathematica program. The quantities of physical interest are the local skin friction coefficient C_{fx} and the local Nusselt number Nu_x are defined as:

$$C_{fx} = \frac{2\tau_w}{\rho U^2} \text{ and } Nu_x = -\frac{q_w x}{k(T_w - T_\infty)}$$

where $\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$ and $q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$ are, respectively, the shear stress and rate of heat-flux at the surface and $U = \upsilon(Gr_x)^{2/5}/x$ is the reference velocity.

Using the transformation (6), then C_{fx} and Nu_x take the following form:

$$\frac{1}{2}(Gr_x)^{1/5}C_{fx} = f''(0)$$

$$\frac{Nu_x}{(Gr_x)^{1/5}} = \frac{1}{\theta(0)}$$

RESULTS AND DISCUSSION

The set of non-linear ordinary differential equations (7) and (8) satisfying the boundary conditions (9) have been solved numerically using the Mathematica method for several values of the involved parameters, namely Prandtl number $\Pr$, pressure work parameter $\in$, heat

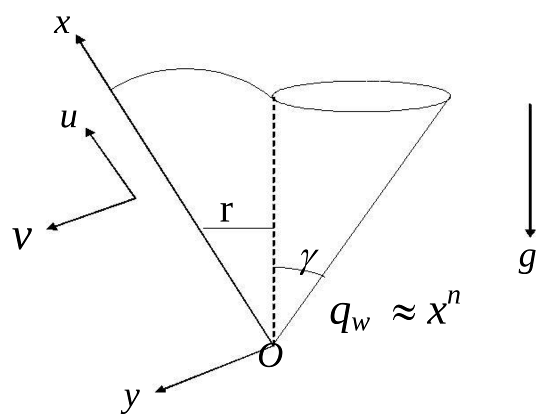

Fig 1: Physical model and coordinates system

Figure 1. Physical model and coordinates system.

The boundary layer equations for steady laminar boundary layer along a vertical cone as [6, 7] are given below.

Table 1. Comparison of Numerical values of local skin-friction and local Nusselt number for various values of Prandtl number $\Pr$ (0.01, 0.05, 0.1) at n = 0.5, $\in = 0$, $\delta = 0$.

$\Pr$	Quantities	Hossain and Paul, 2001	present result
0.01	$f''(0)$	5.13457	5.13320
	$1/\theta(0)$	0.14633	0.14641
0.05	$f''(0)$	2.93993	2.93972
	$1/\theta(0)$	0.26212	0.26224
0.1	$f''(0)$	2.29051	2.29072
	$1/\theta(0)$	0.33174	0.33188

flux gradient parameter n and heat generation/ absorption parameter δ. For validation of numerical method used in this study, the case, when $\in = \delta = 0$ (the pressure work parameter is absent and no heat generation/absorption parameter) has also been considered and the results are compared with those of Hossain and Paul (2001). The quantitative comparison is shown in Table 1 and it is found to be in good agreement with the study of Hossain and Paul (2001)..

The effects of varying Prandtl number $\Pr$, pressure work parameter $\in$, heat flux gradient n and heat generation/absorption parameter δ on the dimensionless velocity profiles $f'(\eta)$ and temperature profiles $\theta(\eta)$ are shown in Figures 2 to 9. Local skin friction coefficient and local Nusselt number are shown in Tables 2 to 4.

Figures 2 and 3 show the dimensionless velocity and dimensionless temperature for Prandtl number $\Pr$ with pressure work parameter $\in = 0.5$, heat flux gradient parameter $n = 0.4$ and heat generation/absorption parameter $\delta = -0.5$. As shown in Figures 2 to 3, the velocity and temperature profiles decreases with effect of Prandtl number $\Pr$. In the case of water at $20^0\,C$ ($\Pr = 7.0$), the free laminar boundary shows a sharp decrease compared to effects in air at $20^0\,C$ ($\Pr = 0.72$). Figures 4 and 5 show the velocity and temperature profiles for pressure work parameter $\in$ with heat flux gradient parameter $n = 0.5$, Prandtl number $\Pr = 0.72$ and heat generation/absorption parameter $\delta = -0.5$. From these figures, it is seen that the velocity and temperature profiles decreases with the effect of pressure work parameter $\in$. From Figures 6 and 7, we observe

δ	-1	0	0.5
$f''(0)$	0.63321	1.02831	1.39302
$1/\theta(0)$	0.96982	0.64295	0.47903

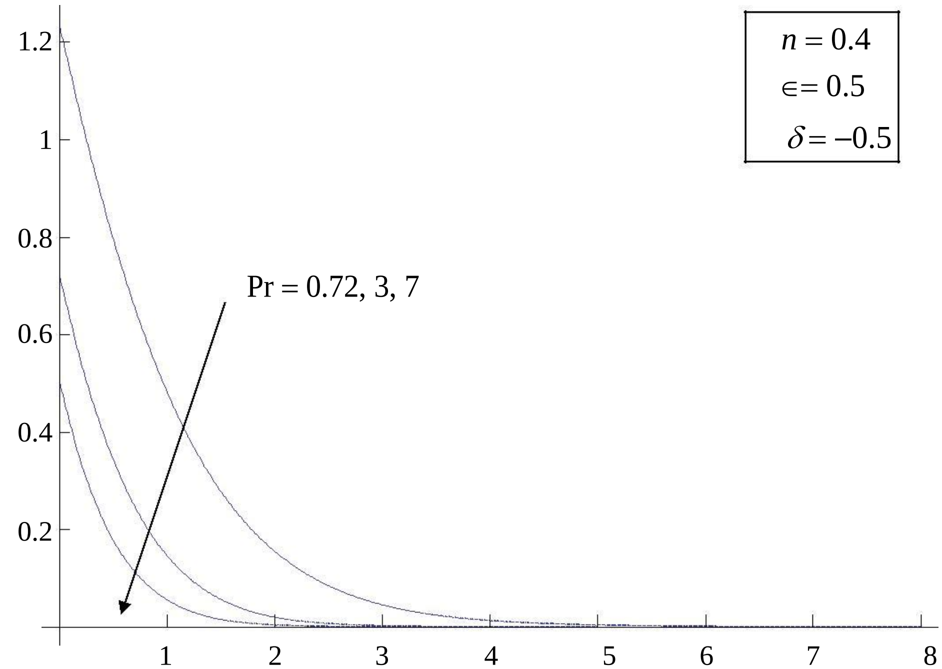

Fig. 2: The temperature profiles $\theta(\eta)$ for various values of Pr at $n = 0.4$, $\delta = -0.5$ and $\in = 0.5$

Figure 2. The temperature profiles $\theta(\eta)$ for various values of Pr at $n =$ 0.4, $\delta = -0.5$ and $\in = 0.5$.

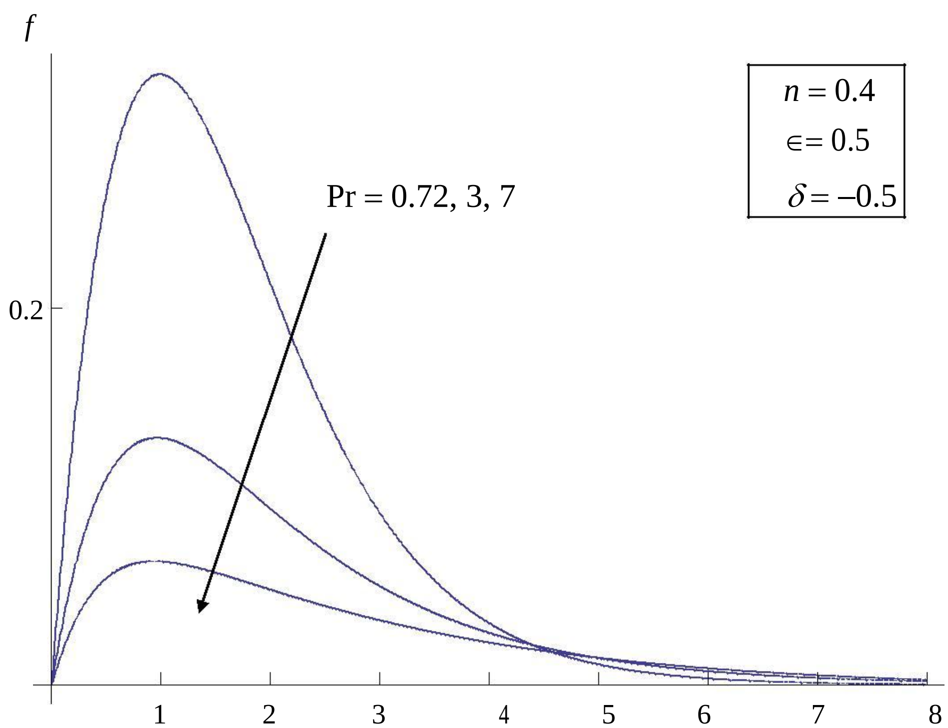

Fig. 3: The velocity profiles $f'(\eta)$ for various values of Pr at $n = 0.4$, $\delta = -0.5$ and $\in = 0.5$

Figure 3. The velocity profiles $f'(\eta)$ for various values of Pr at $n = 0.4$, $\delta = -0.5$ and $\in =$ 0.5.

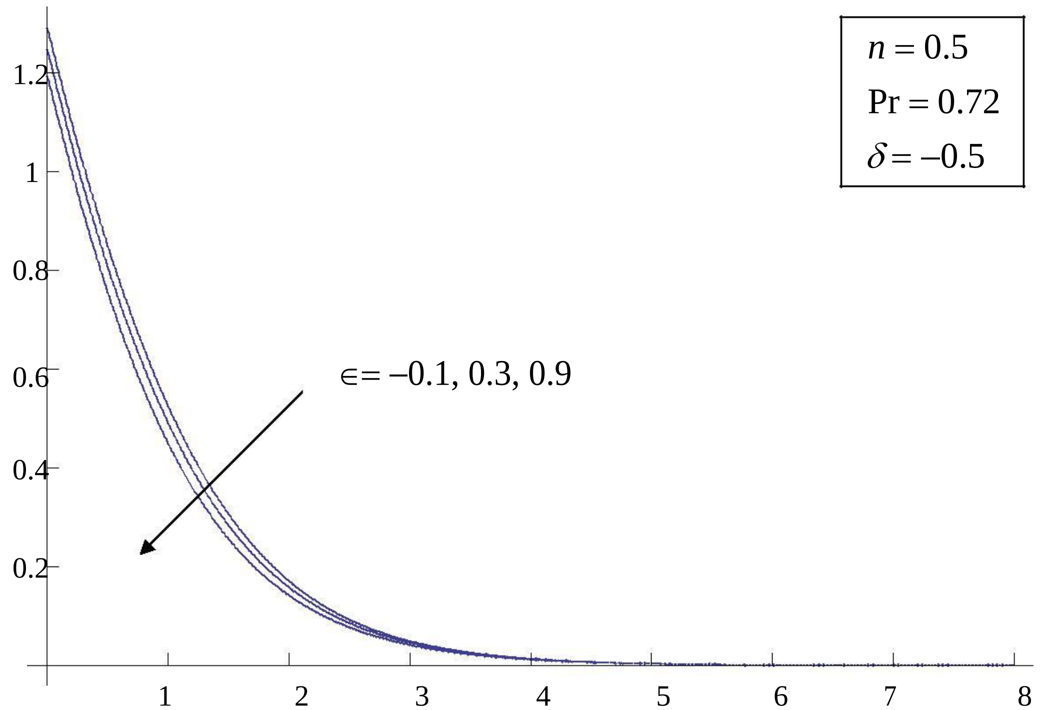

Fig. 4: The temperature profiles $\theta(\eta)$ for various values of $\in$ at $n = 0.5$, $\delta = -0.5$ and $\Pr = 0.72$

Figure 4. The temperature profiles $\theta(\eta)$ for various values of $\in$ at $n = 0.5$, $\delta = -0.5$ and $\Pr = 0.72$.

$n = 0.5$, $\delta = -0.5$ and $\Pr = 0.72$

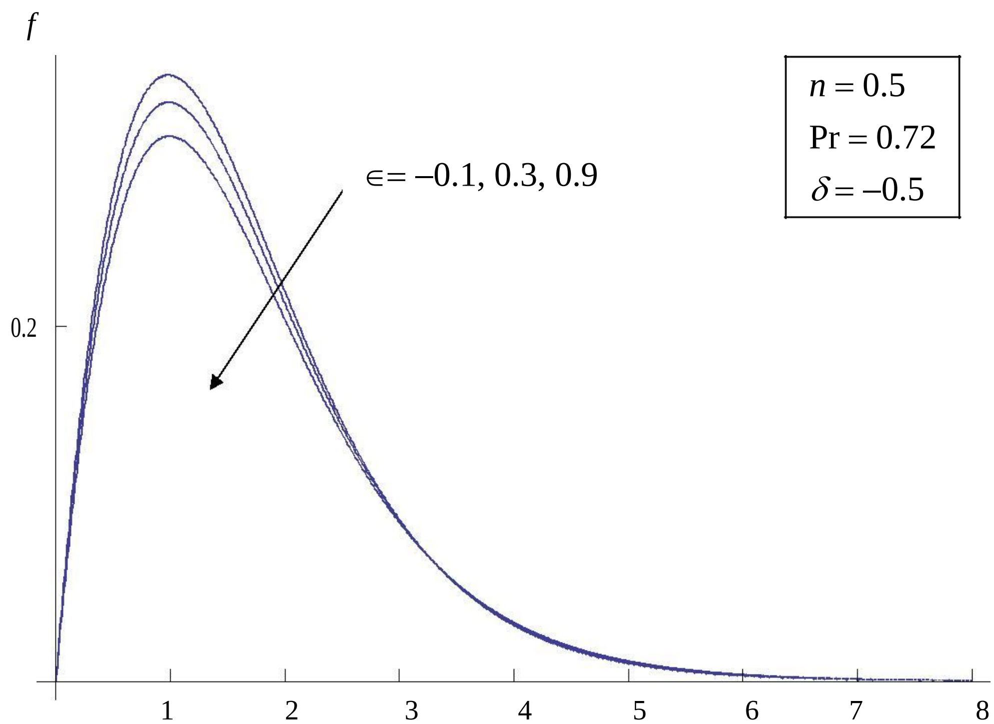

Fig. 5: The velocity profiles $f'(\eta)$ for various values of $\in$ at $n = 0.5$, $\delta = -0.5$ and $\Pr = 0.72$

Figure 5. The velocity profiles $f'(\eta)$ for various values of $\in$ at $n = 0.5$, $\delta = -0.5$ and $\Pr = 0.72$.

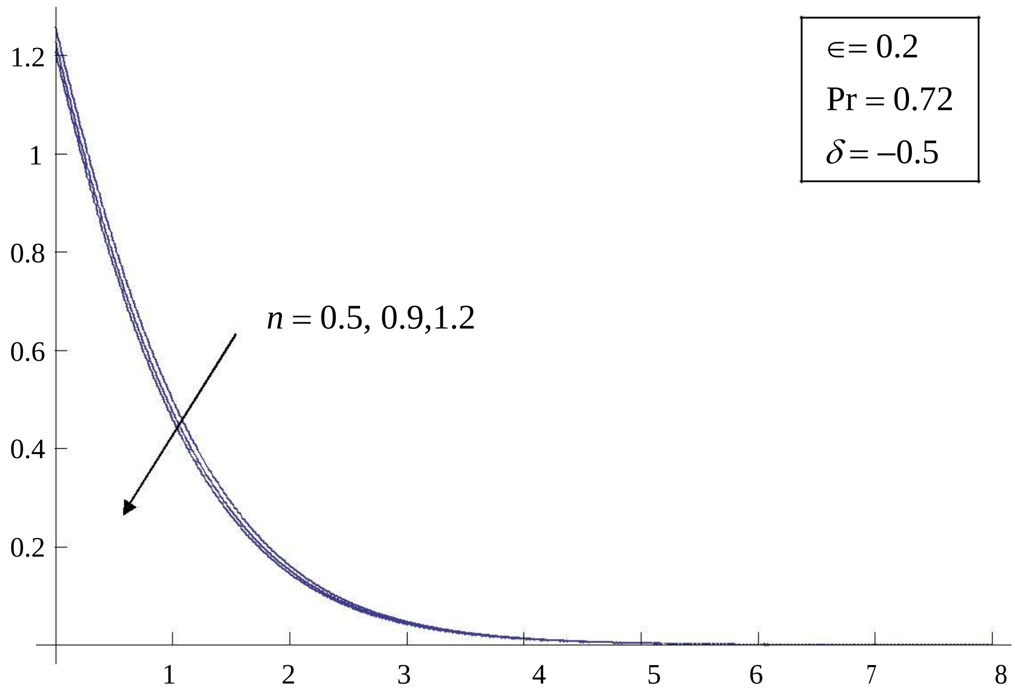

Fig. 6: The temperature profiles $\theta(\eta)$ for various values of n at $\in = 0.2$, $\delta = -0.5$ and $\Pr = 0.72$

Figure 6. The temperature profiles $\theta(\eta)$ for various values of n at $\in = 0.2$, $\delta = -0.5$ and $\Pr = 0.72$.

$\in = 0.2$, $\delta = -0.5$ and $\Pr = 0.72$

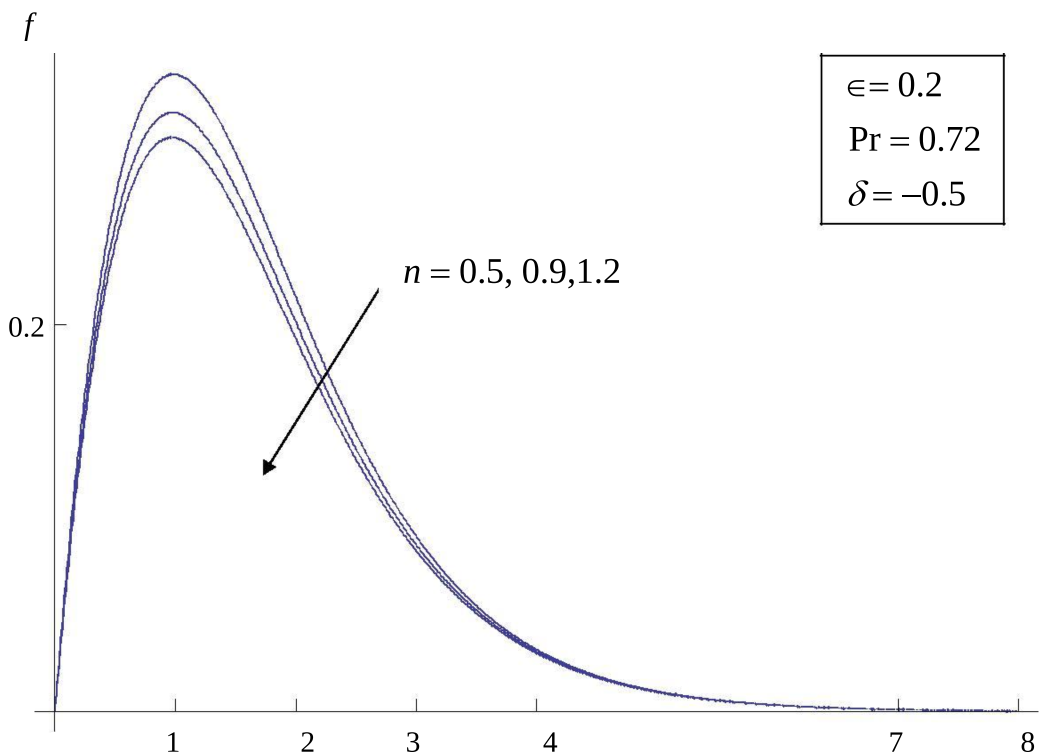

Fig. 7: The velocity profiles $f'(\eta)$ for various values of n at $\in = 0.2$, $\delta = -0.5$ and $\Pr = 0.72$

Figure 7. The velocity profiles $f'(\eta)$ for various values of n at $\in = 0.2$, $\delta = -0.5$ and $\Pr = 0.72$.

$\in = 0.2$, $\delta = -0.5$ and $\Pr = 0$

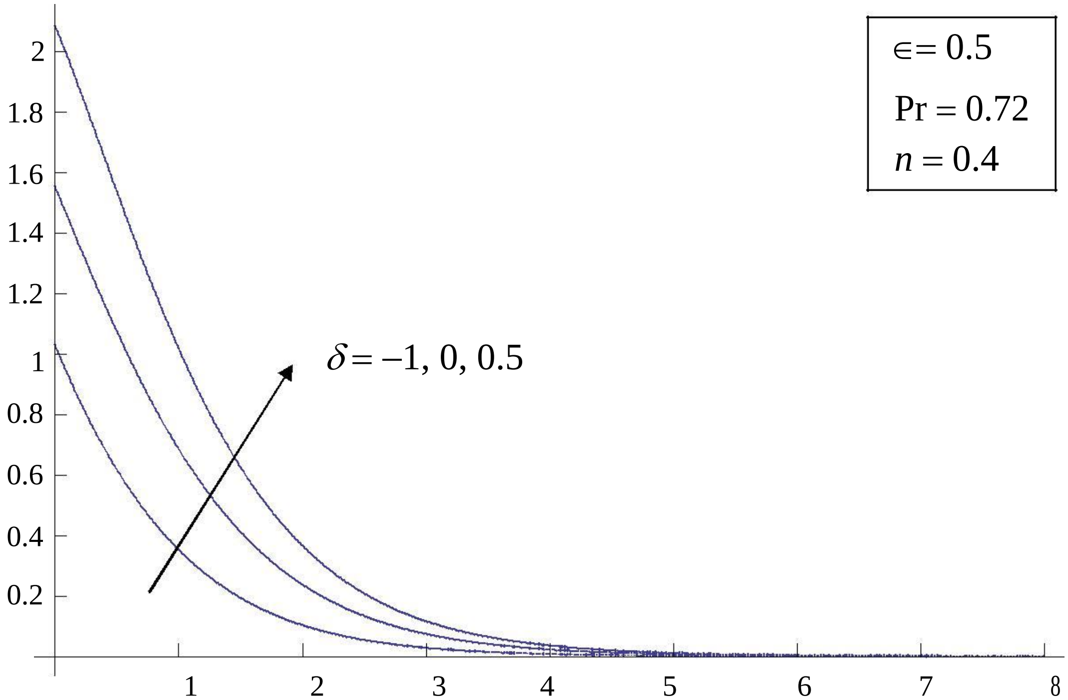

Fig. 8: The temperature profiles θ (η) for various

Figure 8. The temperature profiles $\theta(\eta)$ for various values of δ at $\in = 0.5$, $n = 0.4$ and $\Pr = 0.72$.

values of δ at $\in = 0.5$, $n = 0.4$ and $\Pr = 0.72$

$\in = 0.5$, $n = 0.4$ and $\Pr = 0.72$

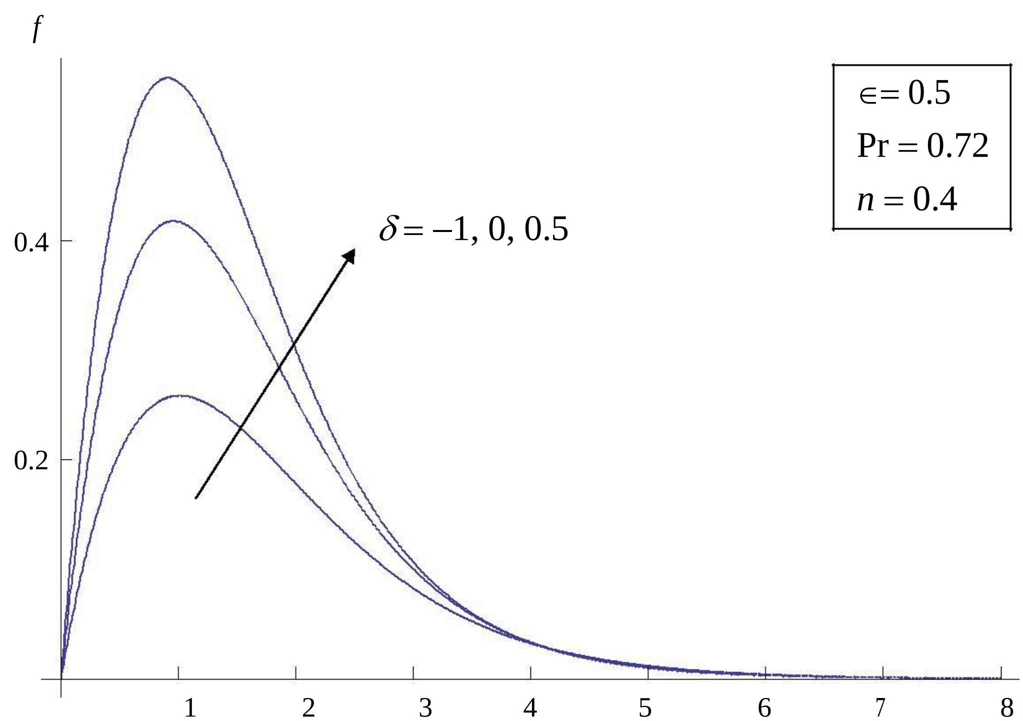

Fig. 9: The velocity profiles $f'(\eta)$ for various

Figure 9. The velocity profiles $f'(\eta)$ for various values of δ at $\in =$ 0.5 , n = 0.4 and $\Pr = 0.72$.

values of δ at $\in = 0.5$, $n = 0.4$ and $\Pr = 0.72$

Table 2. Numerical values of skin-friction and heat transfer coefficient with different values of pressure work parameter $\in$ (-0.1, 0.3, 0.9) at $\Pr$ = 0.72, n = 0.5 and $\delta = -0.5$.

$\in$	**-0.1**	**0.3**	**0.9**
$f''(0)$	0.83373	0.79748	0.75297
$1/\theta(0)$	0.77518	0.80255	0.83842

Table-3. Numerical values of skin-friction and heat transfer coefficient with different values of heat flux gradient n (0.5, 0.9, 1.2) at $\Pr$ = 0.72, $\in$ = 0.2, and $\delta = -0.5$.

n	**0.5**	**0.9**	**1.2**
$f''(0)$	0.80596	0.76908	**0**.74514
$1/\theta(0)$	0.79600	0.81589	0.82939

Table 4. Numerical values of skin-friction and heat transfer coefficient with different values of δ (-1, 0, 0.5) at $\Pr$ = 0.72, $\in$ = 0.5 and $n = 0.4$.

δ	**-1**	**0**	0.**5**
$f''(0)$	0.63321	1.02831	1.39302
$1/\theta(0)$	0.96982	0.64295	0.47903

that the velocity and temperature profiles decreases with the increase of heat flux gradient parameter n with other parameters $\Pr = 0.72$, $\in = 0.2$ and $\delta = -0.5$. Figures 8 and 9, illustrate the effect of heat generation/absorption parameter on the fluid velocity and temperature. It is apparent that the velocity and temperature profiles increase with the increase of heat generation/absorption parameter δ . In Table 2, the effects of pressure work parameter $\in$on the local skin friction coefficient and local Nusselt number are observed. From this Table, it can be seen that an increase in the value of $\in$ leads to a decrease in the values of skin friction coefficient. Also, we observe that the local Nusselt number increase with the increase of the pressure work parameter. In Table 3, the effects of heat flux gradient parameter n on the local skin friction coefficient and local Nusselt number are observed. From this Table, it can be seen that an increase in the value of heat flux gradient parameter n leads to a decrease in the values of skin friction coefficient and increase in the value of local Nusselt number. In Table 4, the effects of heat generation/ absorption parameter δ on the local skin friction coefficient and local Nusselt number are observed. From this table, it can be seen that an increase in the value of heat generation/absorption parameter δleads to increase in the values of skin friction coefficient and decrease in the value of local Nusselt number.

Conclusions

In this paper, the problem of steady, laminar free convection from a vertical circular cone with pressure work and variable surface heat flux in the presence of a heat generation/absorption is studied. From the present investigations, we may conclude the following:

1. The velocity and temperature distribution decreases with increase in the values of $\in$, $\Pr$ and n but an increase in the value of generation/absorption parameter δleads to increase in temperature and velocity profiles.

2. For increasing values of the heat flux gradient n , the skin fiction coefficient decreases, but local Nusselt number increase as the value of the heat flux gradient parameter increase.

3. An increase in the value of Prandtl number $\Pr$ leads to decrease in the value of the skin fiction coefficient, but the local Nusselt number increase as the value of Prandtl number increase.

4. For increase values of the pressure work parameter $\in$, the values of skin fiction coefficient decrease, but the opposite effect is observed for the local Nusselt number.
5. An increase in the value of generation/absorption parameter δ leads to increase in the values of skin friction coefficient and decrease in the value of local Nusselt number.

NOMENCLATURE

C_p Specific heat at constant pressure;
C_{fx} Local skin friction;
f Dimensionless stream function;
g Acceleration due to gravity;
Gr_x The local Grashof number;
k Thermal conductivity of the fluid; n Heat flux gradient;
Nu_x The local Nusselt number coefficient;
Pr Prandtl number;
q_w Surface heat flux;
T Temperature of the fluid; T_w Temperature at the surface;
T_∞ Temperature of the ambient fluid;
u Velocity component in the x- direction;
v Velocity component in the y-direction;
x Measured from the leading edge;
y Distance normal to the surface;

Greek symbols

α The thermal diffusivity;
β Co-efficient of volume expansion;
γ The cone apex half-angle;
η The pseudo-similarity variable;
ν Kinematic viscosity;
μ Viscosity of the fluid;
θ Dimensionless temperature;
ρ Density of the fluid inside the boundary layer.

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