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Full Length Research Paper

The physics facet of price indicator changes: Super-diffusive technique

James Isle*, Eniko Derif and Anibo Parish

Department of Physics, Faculty of Science, Mahidol University, Rama 6 Road, Payathai, Rajathewee, Bangkok, Thailand, 10400

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Attempts were made to quantify the buying and selling interaction of worldwide financial markets into quantitative findings. We introduce a probability density derived from non-extensive Tsallis statistical mechanics that can be applied to the interpretation of percent price index changes for important indices such as NYSE Composite, DJIA, S&P 500, NASDAQ Composite, FTSE 100, NIKKEI 225, Hang Seng, Straits Times and SET index. Results of applying Tsallis' probability density through markets' observation illustrated the behavior of all indices indicating super diffusive dynamics. Furthermore, an Ito-Langevin equation with a time-dependent diffusion coefficient and the nonlinear Fokker–Planck equation can exhibit investment risk of each price index. Finally, we not only explained the complex behavior of financial indices in Physics aspect, but simplified it into quantitative meanings able to be virtually used further as well.

Key words: Econophysics, Tsallis' probability density, Fokker-Planck equation, financial market index.

INTRODUCTION

In financial market, most securities analysts' association consensuses analyze market data by using fundamental statistics, qualitative information and experience making their decisions without exactly understanding their dynamics' type in scientific meaning. Therefore, Physics viewpoints are proposed in order to address this financial market dynamics. This is a kind of application of new interdisciplinary subject in the issue of Econophysics. Though microscopic interactions among traders which lead to behavior in financial markets are not easily understood, there are recently many attempts describing the behavior of market dynamics such as using agent-based model (Amiri and Shirgahi, 2011) and time series analysis (Kamarposhti, 2011). Ising-like model has been a good candidate for describing this behavior for many years (Chowdhury and Stau_erb, 1999; Bornholdt, 2001), but until now, this method has not obviously been the most satisfying model. The advent of non-extensive Tsallis statistical mechanics by means of maximizing the Tsallis entropy (Tsallis, 1988) connected with the essence of the nonlinear Fokker-Planck equation associated with an underlying Ito-Langevin process (Plastino and Plastino, 1995; Borland, 1998) can more specify the type of diffusion and also be a good choice for taking into account of this dynamics. Previous work related to this statistics closely resembled markets' observation including currency exchange price changes (Mantegna and Stanley, 2000), but their algorithms were not able to compare among market indices and were used only well in one hour price change (Michael and Johnson, 2003).

The purpose of this paper is to develop more accurate microscopic interactive traders model based on preceding daily 20 years data in important market indices such as NYSE Composite, DJIA, S&P 500, NASDAQ Composite, FTSE 100, NIKKEI 225, Hang Seng, Straits Times and SET index (Yahoo Finance url: http://finance.yahoo.com.) and compare market risk altogether in at least unit of day. Moreover, investors and others can instantaneously make a comparison of the significant parameters in terms of the risk of investment in a simple way before they can effectively make

*Corresponding author. E-mail: james_isle@hotmail.com, james_isle@gmail.com. Tel: +662-9523007.

investment decisions.

THEORY

Tsallis' non-extensive statistical mechanics was chosen because it could be used to interpret the interactions of a complex system of financial market as follows.

Non-extensive statistical mechanics

Non-extensive Tsallis entropy is written in a different way from statistical mechanics, but it can be proved to be

Boltzmann-Gibbs entropy ($s = -\int p \,\mathrm{LN}\, p$) by taking limit $q \to 1$.

$$S_q = -\frac{1}{1-q}\left(1-\int P(x,t)^q \, dx\right) ;q \in \Re \quad (1)$$

We used constraints for this non-extensive entropy as follows.

$$\int P(x,t)\,dx = 1 \quad (2)$$

$$\left\langle x - \overline{x(t)} \right\rangle \equiv \int \left(x - \overline{x}(t)\right) P(x,t)^q \, dx = 0 \quad (3)$$

$$\left\langle \left(x - \overline{x}(t)\right)^2 \right\rangle_q \equiv \int \left(x - \overline{x}(t)\right)^2 P(x,t)^q \, dx = \sigma_q(t)^2 \quad (4)$$

We found that if q (Tsallis non-extensivity parameter is independent of time) was equal to 1, the 'q-variance' was the ordinary variance. Then, these constraints were maximized by fixed q and got the Tsallis probability distribution function.

$$P(x,t) = \frac{1}{Z(t)}\left\{1 + \beta(t)(q-1)\left(x - \overline{x}(t)\right)^2\right\}^{-\frac{1}{q-1}} \quad (5)$$

where $Z(t)$ is a normalization constant for each time.

$$Z(t) = \frac{B\left(\frac{1}{2}, \frac{1}{q-1} - \frac{1}{2}\right)}{\sqrt{(q-1)\beta(t)}} \quad (6)$$

$$\beta(t) = \frac{1}{2\sigma_q(t)^2 Z(t)^{q-1}} \quad (7)$$

where $B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$ is Euler's Beta function.

The ordinary variance by using distribution of Equation 5 can be derived in Equation 8.

$$\sigma(t)^2 = \left\langle \left(x - x(t)\right)^2 \right\rangle_1 = \int \left(x - \overline{x}(t)\right)^2 P(x,t)\,dx = \begin{cases} \frac{1}{(5-3q)\beta(t)}; & q < \frac{5}{3} \\ \infty; & q \geq \frac{5}{3} \end{cases} \quad (8)$$

Fokker-Planck equation

$$\frac{\partial P(x,t)^{\alpha}}{\partial t} = -\frac{\partial}{\partial x} F(x) P(x,t)^{\alpha} + \frac{D}{2}\frac{\partial^2 P(x,t)^{\nu}}{\partial x^2} \quad (9)$$

$$F(x) = a - bx \quad (10)$$

F(x) is a linear drift force and *D* is diffusion constant.

Condition for solving Tsallis probability distribution function is

$$q = 1 + \alpha - \nu \, . \quad (11)$$

We obtain the following 3 equations dependent of time.

$$-\frac{\alpha}{\alpha+\nu}\frac{dZ^{\alpha+\nu}(t)}{dt} + 2\nu D\beta(t_0) Z(t_0)^{2\alpha} - bZ^{\alpha+\nu} = 0 \quad (12)$$

$$\frac{\beta(t)}{\beta(t_0)} = \left(\frac{Z(t_0)}{Z(t)}\right)^{2\alpha} \quad (13)$$

$$\frac{d\overline{x}}{dt} = a - b\overline{x} \quad (14)$$

μ must be equal to 1 to preserve constraint in Equation 2 by comparing Equations 6 and 13. Then Equations 11 to 13 give

$$\beta(t)^{-\frac{3-q}{2}} = \beta(t_0)^{-\frac{3-q}{2}} e^{-b(3-q)(t-t_0)} - 2Db^{-1}(2-q)\beta(t_0) Z(t_0)^{q-1} \left(e^{-b(3-q)(t-t_0)} - 1\right). \quad (15)$$

Ito-Langevin equation

$$\frac{dx}{dt} = a(x,t) + b(x,t)\,\xi(t) \quad (16)$$

where $\xi(t)$ is δ-correlated Gaussian noise.

$$\left\langle \xi(t)\xi(t') \right\rangle = \delta(t - t') \quad (17)$$

Equation 16 can be proved to be equivalent to Equation 18 (Gardiner, 1997).

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x}\left[a(x,t)P(x,t)\right] + \frac{1}{2}\frac{\partial^2}{\partial x^2}[b^2(x,t)P(x,t)] \qquad (18)$$

Nonlinear Fokker-Planck equation for $\propto = 1$ results from Tsallis probability distribution function.

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x}\left[F(x,t)P(x,t)\right] + \frac{D\partial^2 P(x,t)^{2-q}}{2\partial x^2} \qquad (19)$$

It can be easily seen that

$$a(x,t) = F(x,t) \qquad (20)$$

$$b(x,t) = \sqrt{DP(x,t)^{1-q}}. \qquad (21)$$

Substituting Equations 20 and 21 into Equation 16, we obtain Equation 22

$$\frac{dx}{(t).\, dt} = F(x,t) + \sqrt{DP(x,t)^{1-q}}\,\xi$$
(22)

A theory has been proposed to model anomalous diffusion when it is due to non-Gaussian statistics (Tsallis and Bukman, 1996). This theory led to

$$\alpha = \frac{2}{3-q} \qquad (23)$$

where α is dynamic exponent parameter.

This is the Tsallis relation. This quantifies a connection between anomalous diffusion and non-Gaussian statistics, predicting that a more highly non-Gaussian system (larger q) will exhibit a greater degree of anomalous diffusion (larger q). In other words, particle random motion is predicted to be super diffusive if the probability distribution function is a non-Gaussian distribution with $q > 1$ (Bin et al., 2008). Due to Tsallis relation, we characterized the diffusion types with the *q*-values.

Considering the diffusion coefficient ($DP(x,t)^{1-q}$), first, this is anomalous diffusion correlated in time (memory effects) except for *q* = 1 that is Brownian motion or normal diffusion. Second, if *q* is greater than 1, it is apparently observed that this is super-diffusion and makes the diffusion coefficient large in the next time step. On the contrary, last, if *q* is less than 1, the value of this equation tends to be small jump and that is sub-diffusion.

METHODOLOGY

We applied theory into practice in aspect of behavioral financial market indices. Day-to-day price indices were selected to analyze such as NYSE Composite, DJIA, S&P 500, NASDAQ Composite, FTSE100, NIKKEI 225, Hang Seng, Straits Times and SET index about 20 years from September 1989 to January 2010 (randomly selected period of time), approximately 5,000 days (T=1,2,…,5000).

Each price index is converted into the non-overlapping percent price index change *(x(j))* computed by

$$x(j) = \frac{p(j\cdot t+1) - p((j-1)\cdot t+1)}{p((j-1)\cdot t+1)} \cdot 100 \qquad (24)$$

where *p(T)* is price index at time *T* and *t* is time interval. Fitting (genetic algorithm method) a set of real market index data with Equation 5 which was set an initial time interval (t_0) equal to 1 day, we got an appropriate *q* parameter and $\beta(t_0)$ (Figure 1A). Parameter *b* and *D* were extracted from fitting a set of inverse variance data ($\beta(t)$) with that of its real data from time interval of 1-60 days (Figure 2). Then, Tsallis probability distribution function ($P(x,t)$) from Equation 5 was shown in Figure 1B to E by using the aforementioned inverse variance data ($\beta(t)$) and *q* parameter. Differential Equation 14 was used in order to get a value of parameter a by fitting it with a set of average data of the real percent price index changes increasing with time interval (Figure 3). Diffusion coefficient ($DP(x,t)^{1-q}$) from Equation 22 and from real market data were compared in Figure 4 as well.

RESULTS AND DISCUSSION

There are 5 important parameters such as Tsallis parameter (*q*), dynamic exponent parameter (α), parameters (*a* and *b*) from Equations 10 and 14, and diffusion constant (*D*).

q is greater than 1, which leads to α greater than 1 as shown in Table 1. That means all indices perform the anomalous diffusion in superdiffusive type. In fact, indices should be super-diffusion because the financial market price indices are dependent on each people's decision and interaction among traders. In Figure 1A to E, we depicted the distribution of percent price index changes in each time interval.

It can be seen that the more the time interval increases, the wider the percent price changes distri-butions perform and the lower the highest point of this distributions shows in all indices.

Parameters *a* and *b* from Equation 14 operate the mean's drift or the fluctuation of average percent price index changes. The values of *a* and *b* shown in Table 1 indicate that both differ little from zero. Moreover, the sign of parameter *a* and *b* determine the tendency of mean percent price index changes. That is to say, a positive sign of Equation 14 results in gradual increase in the means of percent price index changes as shown in Figure 3, and the opposite result occurs for a minus sign. Diffusion constant in Equations 9 and 22 plays a significant role in a time-dependent diffusion coefficient ($DP(x,t)^{1-q}$) modeled by Ito-Langevin process.

Therefore, percent price index change distributions

Table 1. The values of essential parameters based on daily price from September 1989 to January 2010.

Parameter	America				Europe	Asia			
	NYSE	DJIA	S&P 500	NASDAQ	FTSE 100	NIKKEI 225	HANG SENG	Straits times	SET
q	1.69	1.60	1.68	1.66	1.40	1.53	1.73	1.59	1.56
α	1.53	1.43	1.52	1.49	1.25	1.36	1.57	1.42	1.39
a	-0.025	-0.026	-0.022	-0.024	-0.015	0.048	-0.031	0.003	0.026
b	0.02	0.017	0.019	0.016	0.015	0.027	0.023	0.014	0.012
D	0.186	0.194	0.193	0.333	0.243	0.602	0.415	0.362	0.559

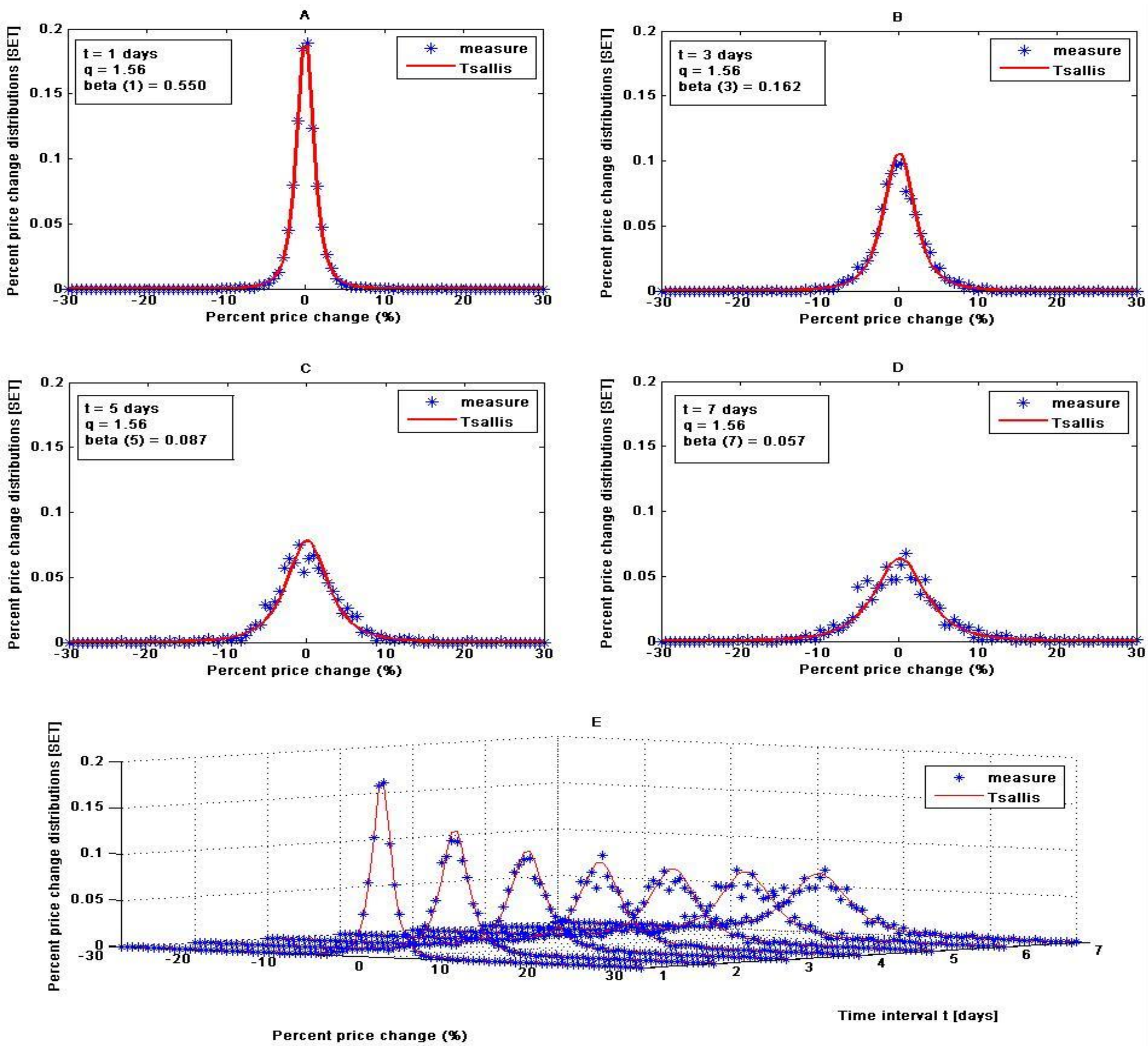

Figure 1. Time evolution of percent price index change distributions only in SET index (A-D) Time interval of 1, 3, 5 and 7 days, respectively (E) Summation of time evolution. The line represents Tsallis data. The star symbol represents the calculated results of real financial market index data.

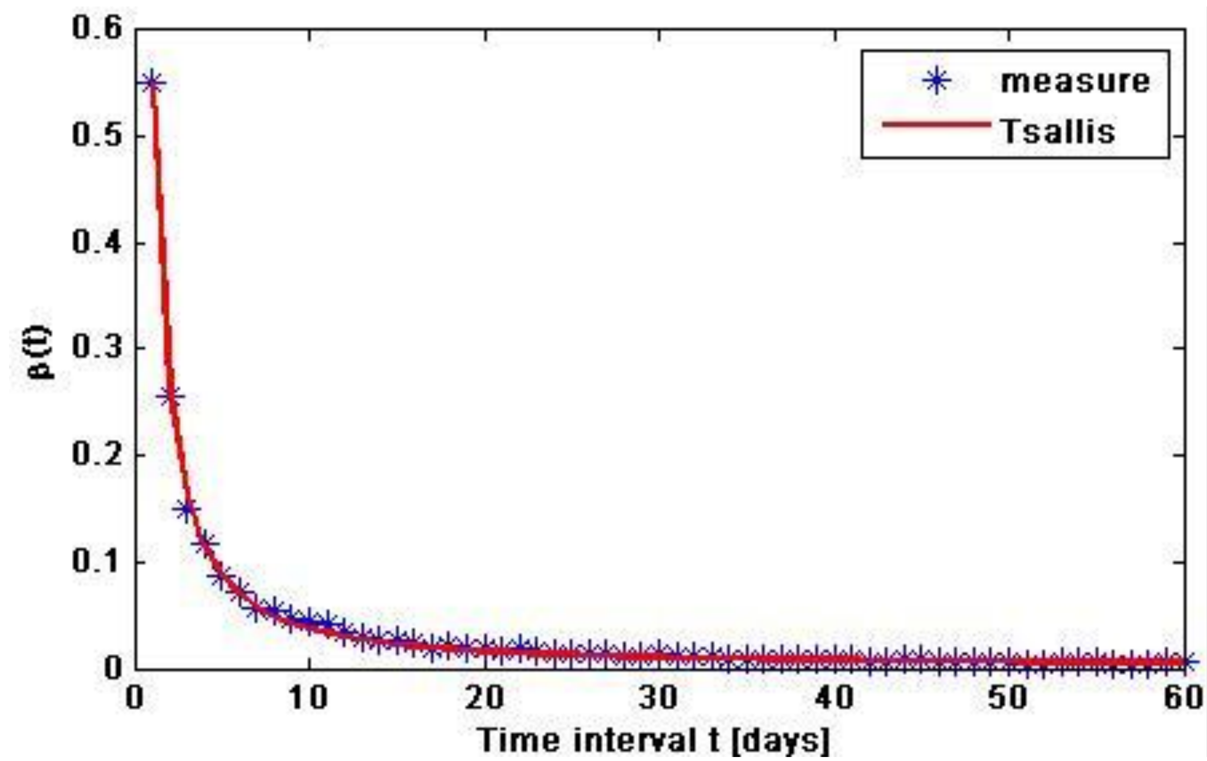

Figure 2. The inverse variance of percent price index changes. The line represents Tsallis data. The star symbol represents the calculated results of real financial market index data.

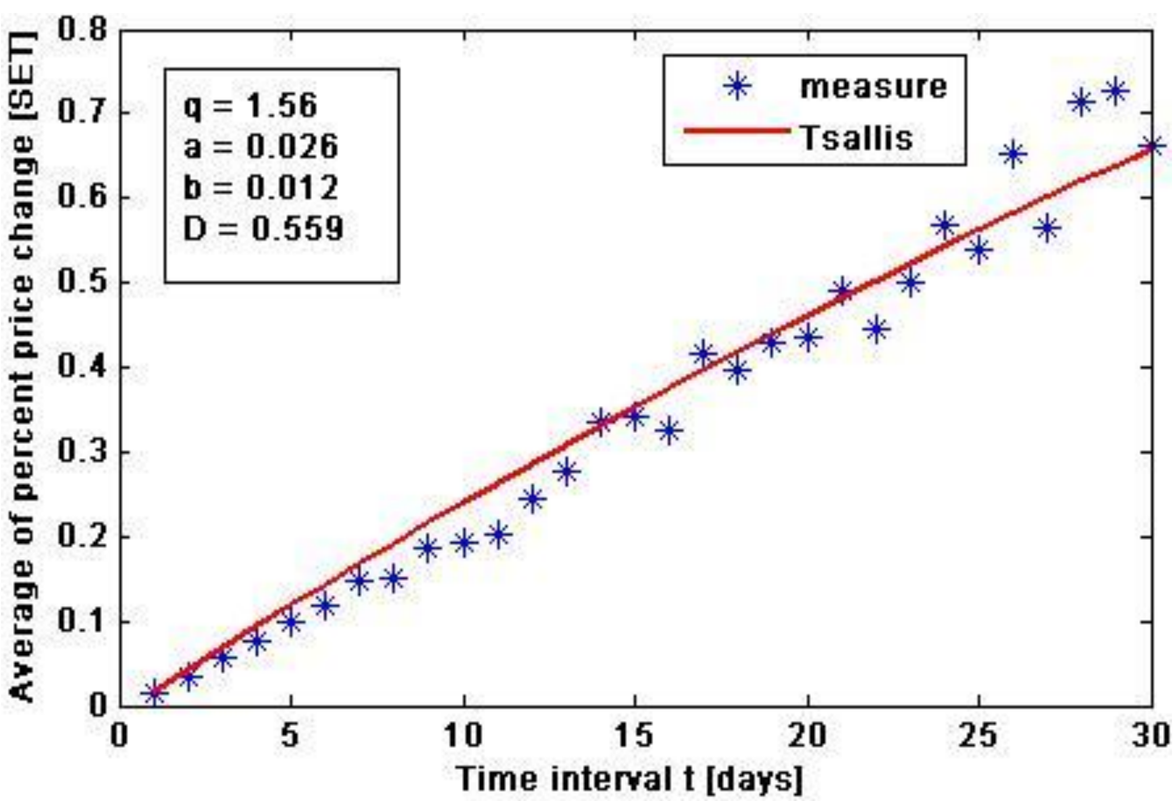

Figure 3. The averages of percent price index changes. The line represents Tsallis data. The star symbol represents the calculated results of real financial market index data.

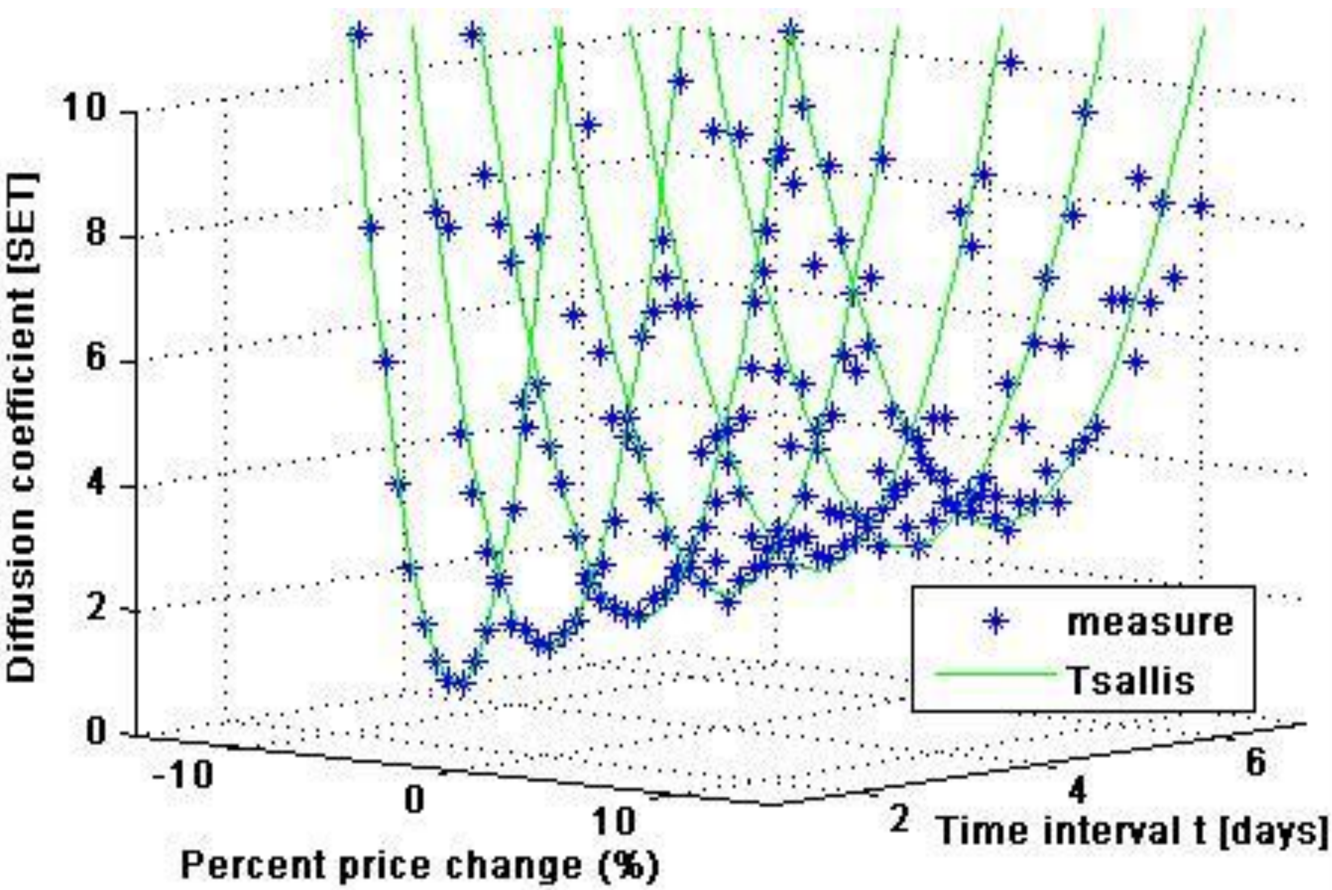

Figure 4. Diffusion coefficient in SET index along with time evolution (time interval from 1 to 7 days). The line represents Tsallis data. The star symbol represents the calculated results of real financial market index data.

depend only on the most recent probability of percent price index changes due to a time-dependent diffusion coefficient and Ito-Langevin equation. We also show time interval evolution of diffusion coefficient in Figure 4. It obviously displayed that if time interval increases, diffusion coefficient mostly decreases at the same percent price index change data. We, however, also found that the lowest diffusion coefficient which is nearly zero percent price index change increases with time interval evolution, that reasonably means the probability to keep price index constant reduces with increasing time interval. The diffusion coefficient can give the investment risk information for investors and others as well. In this case, the recent percent price index change can predict the possibility of percent price index change for the next time interval by using the Ito-Langevin process in Equation 22. This result helps investors to make decisions better whether they should invest in each market.

We show how to use Tables 2 and 3 (some data shown only two indices) to explain the tendency of percent price index changes in a simple way which does not use the Ito-Langevin process in Equation 22. Supposing the recent percent price index change equals -3.33% in the last 1 day time interval, then the diffusion coefficient equals 6.20 in SET index. If DJIA price changes by -3.33% in last 1 day time interval as well, the diffusion coefficient is 4.28. It tells us that percent price index change of SET index will fluctuate more than that of DJIA index, which is proportional to the risk of investment for the next 1 day time interval.

Conclusion

We can explain the behavior of financial market dynamics interpreted by the non-extensive Tsallis distributions connected with time evolution according to a nonlinear Fokker-Plank equation underlying Ito-langevin process with a time-dependent diffusion coefficient indi-cating super-diffusion in all indices. Our results reflect on the interaction among traders in diffusion coefficient term according to the risk of investment that depends on the previous step. We also simplify complicated theory into easier interpretation for commoners by using data from the diffusion coefficient table or the investment risk table.

Table 2. Diffusion coefficient in SET index for each time interval and percent price change.

Diffusion coefficient (SET)		Percent price change (%)																	
		-5.15	**-4.55**	**-3.94**	**-3.33**	**-2.73**	**-2.12**	**-1.52**	**-0.91**	**-0.3**	**0.3**	**0.91**	**1.52**	**2.12**	**2.73**	**3.33**	**3.94**	**4.55**	**5.15**
Time interval (days)	1	12.84	10.31	8.10	6.20	4.62	3.35	2.40	1.76	1.44	1.43	1.73	2.36	3.29	4.54	6.11	7.99	10.2	12.7
	2	8.45	6.97	5.67	4.56	3.63	2.88	2.32	1.94	1.74	1.73	1.91	2.27	2.81	3.53	4.44	5.54	6.82	8.28
	3	6.78	5.73	4.80	4.00	3.34	2.80	2.40	2.12	1.98	1.97	2.09	2.34	2.72	3.23	3.87	4.64	5.54	6.57
	4	5.94	5.11	4.39	3.76	3.24	2.83	2.51	2.29	2.18	2.17	2.25	2.45	2.74	3.13	3.63	4.22	4.92	5.72
	5	5.46	4.78	4.18	3.67	3.24	2.89	2.63	2.44	2.35	2.33	2.40	2.56	2.79	3.11	3.52	4.00	4.57	5.23
	6	5.16	4.58	4.07	3.63	3.26	2.96	2.74	2.58	2.50	2.48	2.54	2.67	2.87	3.14	3.48	3.89	4.37	4.92
	7	4.97	4.46	4.02	3.63	3.31	3.05	2.85	2.71	2.63	2.62	2.66	2.77	2.94	3.17	3.47	3.82	4.24	4.72

Table 3. Diffusion coefficient in DJIA index for each time interval and percent price change.

Diffusion coefficient [DJIA]		Percent price change (%)																	
		-5.15	**-4.55**	**-3.94**	**-3.33**	**-2.73**	**-2.12**	**-1.52**	**-0.91**	**-0.3**	**0.3**	**0.91**	**1.52**	**2.12**	**2.73**	**3.33**	**3.94**	**4.55**	**5.15**
Time interval (days)	1	9.73	7.65	5.84	4.28	2.98	1.95	1.18	0.66	0.41	0.42	0.69	1.22	2.01	3.06	4.37	5.95	7.78	9.88
	2	6.78	5.37	4.13	3.08	2.20	1.50	0.98	0.63	0.47	0.48	0.67	1.04	1.58	2.31	3.21	4.29	5.54	6.98
	3	5.29	4.22	3.28	2.48	1.82	1.29	0.90	0.64	0.52	0.53	0.68	0.97	1.39	1.94	2.64	3.47	4.43	5.53
	4	4.41	3.54	2.79	2.14	1.60	1.18	0.86	0.65	0.56	0.57	0.70	0.93	1.28	1.74	2.30	2.98	3.77	4.66
	5	3.83	3.10	2.46	1.92	1.47	1.11	0.85	0.68	0.60	0.61	0.72	0.93	1.22	1.61	2.09	2.67	3.33	4.09
	6	3.41	2.78	2.23	1.76	1.38	1.07	0.84	0.70	0.63	0.65	0.75	0.93	1.19	1.53	1.95	2.45	3.03	3.70
	7	3.10	2.55	2.06	1.65	1.31	1.04	0.85	0.72	0.67	0.68	0.77	0.93	1.17	1.47	1.85	2.3	2.82	3.41

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