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An economic order quantity model with screening errors, returned cost, and shortages under quantity discounts

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Previous studies on the issue of imperfect quality inventory assumed the direct cost of the product was irrelevant and the screening processes were perfect. However, in practice, the purchase price is some function of the quantity purchased and the inspection testing may fail to be perfect due to Type 1 and Type 2 errors. Thus, this paper proposes a cost-minimizing Economic Order Quantity (EOQ) model that incorporates imperfect production quality, inspection errors (including Type 1 and 2), shortages backordered, and quantity discounts. It is assumed that production, in which 100% screening processes are performed with possible inspection errors, is received with defective quality items and the supplier offers all-unit quantity discounts to the buyer. An algorithm is developed to determine the optimal lot size, shortages and purchase price. Three numerical examples are provided to illustrate the proposed model and algorithm. Numerical computations show that the algorithm is intuitively simple and efficient. Managerial insights are also drawn.

Key words: Inventory, quantity discounts, imperfect quality, screening errors, shortage back-ordering.

INTRODUCTION

The Economic Order Quantity (EOQ) model has been widely used in inventory management for a long time. There are a great many excellent studies that contributed to this topic (for example, Yanasse, 1990; Mehra et al., 1991; Tersine and Barman, 1991; Pantumsinchai, 1991; Min and Chen, 1995; Brill and Chaouch, 1995; Wee, 1993). One unrealistic assumption of the EOQ model is that all units produced are of good quality. Hence, the issue of inventory model with imperfect quality has received considerable attention by researchers. There are a great many papers that have dealt with this topic. Specifically, Rosenblat and Lee (1986) assumed that the defective items could be reworked instantaneously at a cost and found that the presence of defective products motivates smaller lot sizes. At the same time, Porteus (1986) developed a simple model that captures a significant relationship between quality and lot size and observed similar results. Note that the above papers

implicitly assumed the defective items could not be salvaged.

Unlike the assumption of Rosenblat and Lee (1986) and Porteus (1986), Salameh and Jaber (2000) assumed that the defective items could be sold in a single batch at the end of a 100% screening process and found that the economic lot size quantity tended to increase as the average percentage of imperfect quality items increased. Related to this task is the paper by Cardenas-Barron (2000) where an error appearing on Salameh and Jaber's work (2000) was corrected. Thereafter, Chan et al. (2003) proposed a non-shortage model similar to that in Salameh and Jaber (2000), where products are classified as good quality, good quality after reworking, imperfect quality and scrap. With respect to the inventory model proposed in Salameh and Jaber (2000), Chang (2004) fuzzified the defective rate and the annual demand and then derived the corresponding optimal lot sizes.

Papachristos and Konstantaras (2006) re-studied and developed sufficient conditions for the models given by Salameh and Jaber (2000) and Chan et al. (2003). Recently, Wee et al. (2007) and Eroglu and Ozdemir (2007) extended the work of Salameh and Jaber (2000)

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to the case with shortage back-ordering. In their work, they assumed the back-order quantity should be eliminated at the beginning of a replenishment period with "perfect" items in the model. Unfortunately, Wee et al. (2007) neglected the truth and mistakenly employed unscreened items from the received lot to replace the defective items. Furthermore, Maddah and Jaber (2008) employed the renewal process theorem to rectify a flaw in an EOQ model with unreliable supply, characterized by a random fraction of imperfect quality items and a screening process, developed by Salameh and Jaber (2000). Note that the works of Eroglu and Ozdemir (2007) and Maddah and Jaber (2008) are based on the $p \leq 1 - D x /$ assumption. However, Papachristos and Konstantaras (2006) questioned the validity of this assumption, but failed to provide a correction to this defect and pointed out that this condition appearing in Salameh and Jaber's (2000) paper could not guarantee that a shortage will not occur. Fortunately, Jaber et al. (2008) assumed the percentage of defective items per lot according to a learning curve, which was empirically validated by data from the automotive industry. They found that the inspection rate was much higher than the demand rate and with learning effects and the percentage defectives per shipment reduce to a small value.

Lin (2009) further suggested that if the defective rate is within a boundary (for example, if defective rate is with uniform distribution), shortages will not occur under the highly screening rate.

Another unrealistic assumption for imperfect quality inventory models is that the screening processes are perfect. In practice, inspection testing fails to be perfect and two types of errors (Type 1 and 2) may occur. In the first error type, good items will get mis-identified as defectives and thus, result in the necessity of producing additional items. The second error type will result in the mis-identification of defectives as good items and then incur a penalty cost. Lin (2010) considered a single vendor, single buyer supply chain systems in which products are received with defective quality. A 100% screening process is performed with possible inspection errors (Type 1 and 2). Later, Yoo et al. (2009) proposed a profit-maximizing imperfect-quality inventory model with two types of inspection errors (Type 1 and 2) and defective sales return that determines an optimal production lot size.

Note that all previous studies in the topic of inventory models with imperfect quality assumed the direct cost of the product was irrelevant However, in practice; the purchase price is some function of the quantity purchased. Generally, quantity discounts can provide economic advantages for both the buyer and vendor (Burwell et al., 1997; Ji and Shao, 2006). Specifically, quantity discounts can provide the buyer a lower per-unit purchase cost, lower ordering costs and decreased likelihood of shortages (Burwell et al., 1997). Therefore, this paper proposes a cost-minimizing economic order quantity model that

incorporates imperfect production quality, inspection errors (including Types 1 and 2), shortages backordered, and quantity discounts. Specifically, this paper investigates the inventory model for items, where 100% screening processes are performed with possible inspection errors, with imperfect quality and shortage backordering under all-unit quantity discounts.

NOTATION AND ASSUMPTIONS

Following Wee et al. (2007), the notation used in the present paper is in the following:

D The demand rate
K The ordering cost per order
b The backordering cost per unit
d The screening cost per unit
j Discount category
m The penalty cost for replacing one unit of defective returned by the customer.
p The defective percentage for each order
r The holding cost per unit time, expressed as a fraction of dollar value
x The screen rate, $x >> D$
y order size
α Type 1 error; that is, the probability of good items will get mis-identified as defectives
β Type 2 error; that is, the probability of defectives will get mis-identified as good items
c_j The unit purchasing cost of jth level in the cost schedule
$y(c_j)$ The EOQ under the jth level of purchase price
$B(c_j)$ The maximum backordering quantity under the jth level of purchase price
$TC[y(c_j), B(c_j)]$ The total cost per cycle if EOQ is set at y units and backordering quantity is set at B units under jth level of purchase price.
$*$ The superscript representing optimal value.

In all-unit quantity discounts, the discount price applies to all units in the order quantity. Let c_j be the unit price of jth level, y_j be the jth lowest quantity ($y_{j-1} < y_j$). If $y_{j-1} \leq y < y_j$, then the unit price is c_j. The price discount schedule is shown in Table 1.

The assumptions in this paper are as follows:

(1) The demand rate is known, constant, and continuous.
(2) The lead-time is known and constant.
(3) The replenishment is instantaneous.
(4) The screening process and demand proceeds simultaneously, but the screening rate is much higher than demand rate, $x >> D$.
(5) The defective items exist in lot size y in which the

Table 1. Price discount structure.

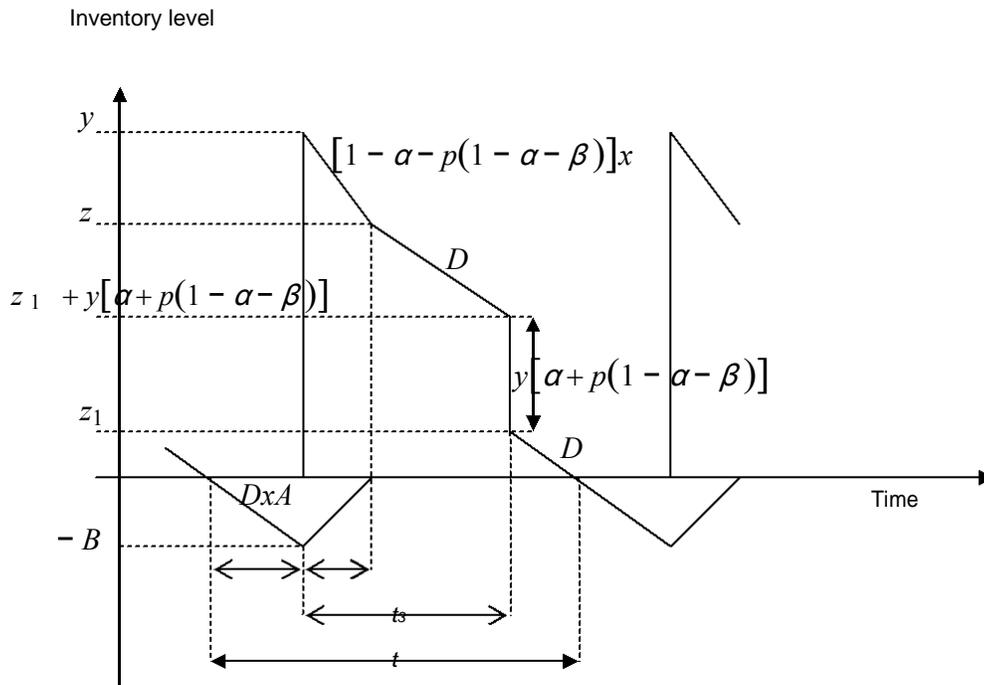| j	$y_{j-1} \leq y < y_j$	c
1	$0 < y < y_1$	c_1
2	$y_1 \leq y < y_2$	c_2
.	.	.
.	.	.
.	.	.
n	$y_{n-1} \leq y < \infty$	c_n

Inventory level

Figure 1. Behaviour of the inventory level over time.

defective percentage p is constant.

(6) Inspection errors may occur in which Type 1 error (α) and Type 2 error (β) are constant.

(7) A shortage is completely backordered and the screening time must be at least or greater than the expected value of the time to eliminate a backorder.

(8) A single product is considered.

(9) A penalty cost is incurred for each defective unit delivered to a customer.

MATHEMATICAL MODEL

Referring to Eroglu and Ozdemir (2007), the inventory level will reduce py unit at time t due to the withdrawal of defective items (Figure 1). However, in this paper, because inspection errors may occur during the 100% screening process, the reduced inventory level should be modified as $\left[py(1-\beta) + (1-p)y\alpha \right]$ unit at time t.

Furthermore, the rate of good-quality items inspected during t_2 is modified as $1 - \alpha - p(1 - \alpha - \beta)$. A part of these good-quality items meet the demand with a rate of D and the remaining is used to eliminate backorders with a rate of $(1 - \alpha - p(1 - \alpha - \beta))x - D = x(1 - \alpha - p(1 - \alpha - \beta) - D/x)$. The screening process is finished at the end of time interval of t_3. To ensure that shortages will not occur, similar to Samalameh and Jaber's (2000) work and Jaber et al.'s (2008) paper, within the screening time t_3, the following condition, in which the screening rate is much higher than the demand rate, should hold:

$$\leq (1 - D/x),\qquad(1)$$

Where, $= \alpha + p(1 - \alpha - \beta)$

Let $TC\left[y(c_j), B(c_j)\right]$ be the total cost per cycle if EOQ is set at y units and the backorder quantity is set at B units under jth level of

purchase price. Note that $TC\left[y(c_j), B(c_j)\right]$ is the sum of the ordering cost, the purchasing cost, the screening cost, the backordering cost, the penalty cost and the holding cost. Since this paper investigates an inventory model for items with imperfect quality and shortage backordering under inspection errors, two types inspection errors may occur: Type 1 and 2 errors. The Type 1 error occurs if perfect items are mistakenly classified as defective, and it results in unnecessarily requiring more items with extra cost. The Type 2 error appears if imperfect items are mistakenly identified as perfect and it incurs a penalty cost. This paper considers the purchase price to be some function of the quantity purchased. Thus, the total cost can be written as follows:

$$TC\left[y(c_j), B(c_j)\right] = K + c_j y + dy + \left[bB(t_1 + t_2)/2\right] + py\beta m$$

$$+c_j \frac{t_2 (y+z) + (t_3 - t_2)[z+z_1 + y] + (t - t_1 - t_3)z_1}{2}$$

$$j = 1,2,..., n \tag{2}$$

Now, referring to Figure 1, there are some findings shown below:

The time, t_1, needed to add up to a backorder level of 'B' units is given by

$$t_1 = B'/D, \tag{3}$$

The time, t_2, needed to eliminate the backorder level of 'B' units under screening error consideration can be written as:

$$t_2 = B'/(xA), \tag{4}$$

Where, $A = 1 - (D/x)$.

By Equations (3) and (4), the value of z can be written as:

$$z = y - \frac{(1-)B}{A} \tag{5}$$

The time, t_3, used to screen the ordered units, y, in each cycle is

$$t_3 = y/x \tag{6}$$

Thus, the value of $(t_3 - t_2)$ in Figure 1 is given by

$$t_3 - t_2 = \frac{z - z_1 - y}{D} \tag{7}$$

Furthermore, by Equations (6) and (7), the value of z_1 can be expressed as follows:

$$z_1 = Ay - B \tag{8}$$

Substituting Equations (3) - (8) into Equation (2), one has

$$TC\left[y(c_j), B(c_j)\right] = (c_j \qquad \frac{c_j r}{2} \frac{2}{x} \frac{D x(1 - D x)^2}{D}$$

$$-\frac{c_j r(1-)By(c_j r + b)(1-)B^2}{D} + \frac{}{2D(1 - - D/x)}, \; j = 1,2,...,n \tag{9}$$

Since the replenishment cycle length is $t = y(1-)/D$, the total cost per unit time can be given as follows:

$$TCU\left[y(c_j), B(c_j)\right] = \frac{D(c_j + d + \beta mp)}{\Omega} + \frac{DK}{y\Omega 2\Omega} + \frac{c_j ry\theta}{} - c_j rB + \frac{(c_j r + b)B^2}{2y(1 - - D x)},$$

$$j = 1,2,...,n \tag{10}$$

Where, $\Omega = 1 -$, $\theta = \frac{D(2 - D/x)}{x} + \left[(1 - - D x)^2\right]$.

Note that if the supplier does not offer quantity discounts and all items are perfect (that is, no screening process is needed), the study has $t = d = \alpha = \beta = p = 0$, $x \to \infty$, and $c_j = c$. Therefore, in this case, Equation (10) is reduced to the classical EOQ model with shortages.

SOLUTION PROCEDURE AND ALGORITHM

To find the optimal lot size and the optimal backordering period under all-unit quantity discounts, let y be fixed. Taking the first and second derivatives of $TCU\left[y(c_j), B(c_j)\right]$ with respect to B will yield:

$$\frac{\partial TCU\left[y(c_j), B(c_j)\right]}{\partial B} = -c_j r + \frac{(c_j r + b)B}{y(1 - - D/x)}, \; j = 1,2,...,n \tag{11}$$

$$\frac{\partial^2 TCU\left[y(c_j), B(c_j)\right]}{\partial B^2} = \frac{(c_j r + b)}{y(1 - - D x)} > 0, \; j = 1,2,...,n \tag{12}$$

Since the screening rate is much higher than demand rate and thus $\leq (1 - D/x)$, $TCU\left[y(c_j), B(c_j)\right]$ is convex in B. Let $dTCU\left[y(c_j), B(c_j)\right]/dB = 0$. The study has

$$B(c_j) = \frac{c_j ry(1 - - D/x)}{(c_j r + b)}, \; j = 1,2,...,n \tag{13}$$

Substituting Equation (13) into Equation (10) will yield

$$TCU\left[y(c_j)\right] = \frac{D(c_j + d + \beta mp)}{\Omega} + \frac{DK}{y\Omega 2\Omega} + \frac{c_j ry\theta}{} - \frac{y(c_j r)^2(1 - - D/x)}{2(c_j r + b)}$$

$$, j = 1,2,...,n \tag{14}$$

Taking the first and second derivatives of $TCU\left[y(c_j)\right]$ with respect y leads to

$$\frac{\partial TCU\left[y(c_j)\right]}{\partial y} = \frac{-KD}{y^2 \Omega} + \frac{c_j r\theta}{2\Omega} - \frac{(c_j r)^2(1 - - D/x)}{2(c_j r + b)}, \; j = 1,2,...,n \tag{15}$$

$$\frac{\partial^2 TCU\left[y(c_j)\right]}{\partial y^2} = \frac{2KD}{y^3 \Omega} > 0, \; j = 1,2,...,n \tag{16}$$

From Equation (16), we know $TCU\left[y(c_j)\right]$ is convex in y. Since the purchasing unit cost depends on the order quantity and thus has a different cost curve, we cannot directly obtain y^* from Equation (15).

Thus, let $\tilde{y}_j$ be the lowest point on each cost curve c_j. By setting Equation (15) equal to zero, one has

$$\tilde{y}_j = \sqrt{\frac{2KD(c_j r + b)}{(c_j r\theta)(c_j r + b) - (c_j r)^2 \Omega\left(1 - \frac{D}{x}\right)}}, \quad j = 1,2,...,n \quad (17)$$

It is clear that the denominator is greater than zero. To obtain the optimal solution, an algorithm, similar to Goyal (1995) and Lin (2010), is developed as follows:

Step 1. Obtain the possibly maximum unit cost c^* as follows:

$$c^* = \frac{1}{D\Omega}\left[Dc_n + \frac{DK}{y_{n-1}} + \frac{c_n r y_{n-1}\theta}{2} - \frac{\Omega}{2(c_n r)^2}\frac{y_{n-1}^2(1 - D/x)}{2(cr+b)}\right]$$

$$- \sqrt{\frac{2KD\left[(c_n r\theta)(c_n r + b) - (c_n r)^2 \Omega(1 - D/x)\right]}{c_n r + b}}$$

Step 2. For each purchase price discount $c_n \le c_j < c^*$, determine the total relevant cost $TCU\left[y(c_j), B(c_j)\right]$ from Equation (14) at order size given by

$$y(c_j) = \text{MAX}\left\{y_{j-1}, \sqrt{\frac{2KD(c_j r + b)}{c_j r\theta(c_j r + b) - (c_j r)^2 \Omega(1 - D/x)}}\right\}$$

and at shortage backordering given by

$$B(c_j) = \frac{c_j r y(1 - D/x)}{c_j r + b},$$

where $y = y(c_j)$.

Step 3. Compare all costs obtained in Step 2. The lowest relevant cost provides the optimal order quantity, $y(c_j)$ and optimal shortage backordering, $B(c_j)$.

In the next section, the study will employ the example to illustrate the model and algorithm developed in Sections 3 and 4, respectively.

NUMERICAL EXAMPLES

To illustrate the above model and algorithm, consider the following examples.

Example 1

The parameters needed for analyzing the models developed in this paper are thus given:

Demand rate, D =50000 units/year,
Ordering cost, K =$100/cycle,
Screening rate, x =1 unit/min
Screening cost, d =$0.5/unit
Backordering cost, b =$20/unit/year
Penalty Cost, m = $100/unit
Defective rate, p = 0.02
Type 1 error, α = 0.01
Type 2 error, β = 0.01
A percentage of unit purchase cost, r =0.2

Assume that the supplier offers a purchase price discount schedule as shown in (Table 2). Considering the cost structure category and using the algorithm developed in the work, the study can obtain the optimal order quantity, shortages and minimum cost.

Step 1. Obtain the possibly maximum unit cost as follows: $c^* = \$23.707 / item$

Step 2. Consider the condition of purchasing price break segment; there are two scenarios, $c_5 = 23$ and $c_4 = 23.5$, satisfying $23 \le c_j < 23.707$. One has

$y(c_4 = 23.5) = \text{MAX}[1500, 1588.96] = 1588.96$;
$y(c_5 = 23.0) = \text{MAX}[2000, 1604.19] = 2000$ and
$B(c_4 = 23.5) = 193.95$; $B(c_5 = 23.0) = 239.89$

Thus, the $TCU\left[y(c_j), B(c_j)\right]$ from Equation (10) at order and shortage quantities is given by

$TCU[y(c_4 = 23.5), B(c_4 = 23.5)] = TCU(1588.96, 193.95) = 1244119$

$TCU[y(c_5 = 23.0), B(c_5 = 23.0)] = TCU(2000, 239.89) = 1218452$

Step 3. Since

$TCU[y(c_4 = 23.5), B(c_4 = 23.5)] > TCU[y(c_5 = 23.0), B(c_5 = 23.0)]$,

one has the optimal order quantity, $EOQ = 2000$, the shortage backordering, $B = 239.89$, and minimum total relevant cost, TCU=1218452, under the purchasing cost of $c_5 = 23.0$.

Example 2

If D is changed to 15,000 units/year, K is changed to $300/cycle, and the supplier offers another price discount schedule as follows (Table 3).

The same procedures are performed as Example 1. Thus, the study has the optimal order quantity, $EOQ = 1546.41$, the shortage backordering, $B = 261$, and minimum total relevant cost, $TCU = 385171$, under the

Table 2. Purchase price discount structure.

j	$y_{j-1} \sim y_j$	c
1	$0 < y < 500$	$c_1 = 25.0$
2	$500 \le y < 1000$	$c_2 = 24.5$
3	$1000 \le y < 1500$	$c_3 = 24.0$
4	$1500 \le y < 2000$	$c_4 = 23.5$
5	$y \ge 2000$	$c_5 = 23.0$

Table 3. Purchase price discount structure.

j	$y_{j-1} \sim y_j$	c
1	$0 < y < 500$	$c_1 = 24.04$
2	$500 \le y < 1000$	$c_2 = 24.03$
3	$1000 \le y < 1500$	$c_3 = 24.02$
4	$1500 \le y < 2000$	$c_4 = 24.01$
5	$y \ge 2000$	$c_5 = 24.00$

purchasing cost of $c_4 = 24.01$.

Example 3

If all parameters are same as Example 2 except defective rate is changed to 0.03, the optimal solution will leads to (1) the optimal order quantity, $EOQ = 1560$ (2) the shortage backordering, $B = 260$ (3) minimum total relevant cost, TCU= 389204 (4) the unit purchasing cost, $c_4 = 24.01$.

CONCLUSIONS

In this paper, an economical order quantity model with imperfect quality and shortage backordering under inspection errors and quantity discounts was developed. Since the optimal solution should simultaneously determine the lot size, shortages, and purchase price, we tactfully restructured the mathematical model using the relationship between the order quantity and shortages. To find the optimal solution, an algorithm is further built. Numerical computations show that our algorithm is intuitively simple and efficient, in which it requests fewer iterations.

Numerical results also revealed that (1) the lowest unit purchasing cost may not guarantee to obtain the minimum total relevant cost (2) the policy of quantity discounts has significantly influence on the optimal solution (3) the more defective rate, the more total relevant cost is (4) if defective rate increases, the order quantity increases while shortages decreases under the same unit purchasing cost. Furthermore, when all items are perfect

(that is, no screening process is needed) and the supplier does not provide quantity discounts (that is, $c_j = c$), the model developed in this study reduces to the classic economic order quantity model with shortages.

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